

Multiple car ownership, fuel efficiency and substitution between cars

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Abstract

In this paper we develop a microeconomic model of multiple car ownership, fuel efficiency of the car(s) owned, and car use. These three important aspects of the demand for cars are studied in a simultaneous choice framework. The framework allows us to investigate how fuel prices affect car ownership decisions and the optimal choice of fuel efficiency, how exogenous parameters affect relative car use by first and second cars in the household, etc. Some implications of the analysis are illustrated using a simulation version of the model, focusing on the substitution between cars in multiple vehicle households and on the role of fuel efficiency choices. We analyze the impact of fuel prices and the fixed cost structure of cars of different fuel efficiency for car ownership decisions, choice of fuel efficiency, demand for car travel and overall fuel use.

Introduction

The purpose of this paper is to study the long-run choices of households with respect to car ownership, the quality of the cars selected, and the associated demand for kilometers driven by the various cars. We focus on a particular aspect of quality that is highly relevant in view of environmental policies, viz., fuel efficiency. The model we develop allows for potential substitution between cars in multiple vehicle households. In such a setting, the question is how the number of cars owned, the fuel efficiency of the vehicles selected, and the demand for kilometers by different cars will in the long run be affected by the fixed (purchase prices, taxes) and variable cost (fuel prices) structure of car travel demand.

Of course, there is a large theoretical and empirical discrete choice literature dealing with households' choice of the number and type of cars to own, and the associated demand for car kilometers. Among many others, Mannering and Winston (1985), Train (1986) and De Jong (1990, 1991) developed detailed empirical analyses of car ownership and car use in both single and multiple vehicle households. More recently, Fullerton and Gan (2005) and Feng, Fullerton and Gan (2005) analyze empirical models that allow simultaneous estimation of decisions related to car ownership, car type and kilometre demand, and they use the models to study the relative efficiency of different emission reduction policies (emission tax, fuel tax, annual registration fees, etc.). However, all these models typically either ignore substitution effects between cars in multiple car households, or deal with such possibilities in an ad hoc way. This is somewhat surprising because, in an early study, Mannering (1983) did allow for substitution possibilities in an econometric analysis of car use in households with multiple vehicles, and indeed found significant substitution effects between cars. More recently, Golob and McNally (1997) and Golob, Kim and Ren (1996) similarly emphasized travel interactions within households. Finally, a small recent literature focusing on issues of optimal taxation of different types of cars (e.g., diesel versus gasoline) has limited the analysis to single car owners (see, e.g., Chia, Tsui and Whalley (2001), De Borger and Mayeres (2007)).

In this paper, we develop a theoretical model of households' optimal choices of the number of cars to own, the fuel efficiency of the cars selected, and the demand for

kilometers driven by the different cars owned. Implications are illustrated using a simulation version of the model, focusing on the substitution between cars in multiple vehicle households and the role of fuel efficiency choices¹. We analyze the impact of fuel prices and the fixed cost structure of cars of different fuel efficiency for optimal choices. This allows us to investigate how fuel prices affect car ownership decisions and choice of fuel efficiency, what the implications are for total travel demand and for total fuel consumption, how exogenous parameters affect relative car use by first and second cars in the household, etc.

The paper is structured as follows. We describe the setup of the basic model in Section 1. The next sections then first explore households' optimal long-run choices of fuel efficiency and demand for kilometers, conditional on car ownership decisions. In Section 2, we analyze this problem conditional on the choice to own a single car. Section 3 studies the joint decisions of fuel efficiencies and kilometer demands, conditional on households having two cars. Section 4 analyzes the optimal demand and fuel efficiency choice of a household owning one car and deciding to buy a second one: how does the fuel efficiency of the car owned affects the fuel efficiency of the new car bought? Having gained insight into households' conditional decision making, Section 5 looks at the discrete choice of how many cars to own, and it studies potential implications of fuel prices for car ownership, for fuel efficiency, for total fuel use, and for total car travel demand. The role of potential substitution between cars is explored. In Section 6 we provide a simulation exercise illustrating the model. Conclusions are summarized in Section 7.

¹ The model studied in this paper is also related to two other strands of recent literature. One is the literature on aggregate but separate effects of policy parameters on the vehicle stock, fuel efficiency and energy use (Johansson and Schipper (1997), Small and Van Dender (2006)). Another link is to the literature on multiple discreteness that has recently received substantial attention. See in particular Bhat (2005, 2008). These models were originally developed to deal with consumers who use varying amounts of different types of nondurable consumption goods or participate in various activities for time spells of varying length, but there have also been some applications to automobile demand, see Bhat and Sen (2006) and Bhat (2008).

1. Structure of the model

We limit the analysis to a maximum of two cars per household; extension to more cars is conceptually straightforward. Let us denote the demand for kilometers by q_1, q_2 for the first and second cars, respectively. We will define the first car below as the one with the largest marginal utility for the first kilometer driven; more details follow. We denote car i 's fuel efficiency (quality) as k_i ; it gives the distance a car can travel per liter of fuel consumed. However, to facilitate the derivations below, it will be instructive to also define inverse fuel efficiency (fuel consumption per unit of distance); this will be denoted by $g_i = \frac{1}{k_i}$.

Since this paper focuses on fuel efficiency, we assume fuel is the only variable cost. The variable fuel price per kilometer driven by car i is then defined as

$$p_i = \pi g_i$$

The variable cost is an increasing function of the fuel price π per liter (a decreasing function of its 'quality'). Our focus on fuel efficiency as the only quality aspect implies a 'simple repackaging' (see Fisher and Shell, 1968) formulation of quality. However, unlike virtually all other models in the literature, our focus is on a durable consumption good. This means that we also have to take into account the fixed annual ownership cost of the car. We assume that this ownership cost also depends on inverse fuel efficiency; it is denoted by a function

$$f(g_i), \quad \frac{\partial f(g_i)}{\partial g_i} < 0; \quad \frac{\partial^2 f(g_i)}{\partial g_i^2} > 0$$

More fuel efficient cars have a higher annual ownership cost. The marginal ownership cost of further improvements in fuel efficiency rises in fuel efficiency. We assume that all cars are identical except for fuel efficiency and that a wide range of fuel efficiencies is available on the market (new cars, second-hand markets).

Apart from car kilometers q_1, q_2 , assume the household cares for a general consumption good x (its price is normalized at one). We can then denote the utility function of the household as $u(x, q_1, q_2)$. In the long-run, the household can optimally choose how many cars to own (zero, one, or two), it can select the cars according to their

most desired fuel efficiency, and it can decide on the optimal consumption of the numeraire good and car kilometers.

In such a setting, it is customary to model the discrete choice of how many cars to own by comparing utilities, conditional on the number of cars owned. Denoting the conditional utilities by w^i ($i = 0, 1, 2$), they are defined by the solution to the following conditional optimization problems:

$$w^0(y) = u(y, 0, 0) \quad \left(= \underset{x}{\text{Max}} \quad u(x, q_1, q_2) \quad \text{s.t.} \quad q_1 = 0, q_2 = 0, x = y, \right)$$

$$w^1(y, \pi) = \underset{x, q_1, g_1}{\text{Max}} \quad u(x, q_1, q_2) \quad \text{s.t.} \quad q_2 = 0, x + \pi g_1 q_1 = y - f(g_1)$$

$$w^2(y, \pi) = \underset{x, q_1, q_2, g_1, g_2}{\text{Max}} \quad u(x, q_1, q_2) \quad \text{s.t.} \quad x + \pi g_1 q_1 + \pi g_2 q_2 = y - f(g_1) - f(g_2)$$

Comparison of the w^i ($i = 0, 1, 2$) and selecting the largest utility solves the discrete choice problem. The continuous choices of fuel efficiency and kilometer demand follow from the relevant conditional optimization problem, given the choice of the number of cars.

Alternatively, however, one can think of both the discrete and the continuous choices to be the solution of the same general optimization problem the household faces. All choices with respect to the number of cars, fuel efficiency and kilometers driven are then the result of maximizing $u(x, q_1, q_2)$ under various constraints. In its most general form, the household's long-run problem can then be formulated as:

$$\begin{aligned} \underset{x, q_1, q_2, g_1, g_2}{\text{Max}} \quad & u(x, q_1, q_2) \quad \text{s.t.} \quad x + \pi g_1 q_1 + \pi g_2 q_2 + f(g_1) + f(g_2) \leq y \quad (\lambda) \\ & g_1 - \hat{g} \leq 0 \quad (\delta_1) \\ & g_2 - \hat{g} \leq 0 \quad (\delta_2) \\ & q_1 \geq 0, q_2 \geq 0, g_1 \geq 0, g_2 \geq 0 \end{aligned}$$

Here $\lambda, \delta_1, \delta_2$ are the non-negative multipliers associated with the weak inequality constraints. In the above formulation, note that a hypothetical inverse fuel efficiency level \hat{g} is introduced to guarantee that, if it is optimal for the consumer to drive zero kilometers with car i , fixed annual spending on this car type will also be zero. Fuel efficiency \hat{g} is that level of inverse fuel efficiency that results in a zero annual fixed cost, $f(\hat{g}) = 0$. Literally, a car with fuel efficiency \hat{g} has such extreme fuel consumption

that it can be obtained at a zero price on the market. To see the implications of this formulation, note that the first-order conditions are

$$\begin{aligned}
\frac{\partial u}{\partial x} - \lambda &\leq 0; & \left(\frac{\partial u}{\partial x} - \lambda \right) x &= 0 \\
\frac{\partial u}{\partial q_1} - \lambda \pi g_1 &\leq 0; & \left(\frac{\partial u}{\partial q_1} - \lambda \pi g_1 \right) q_1 &= 0 \\
\frac{\partial u}{\partial q_2} - \lambda \pi g_2 &\leq 0; & \left(\frac{\partial u}{\partial q_2} - \lambda \pi g_2 \right) q_2 &= 0 \\
-\pi q_1 - \frac{\partial f(g_1)}{\partial g_1} - \delta_1 &\leq 0; & \left(-\pi q_1 - \frac{\partial f(g_1)}{\partial g_1} - \delta_1 \right) g_1 &= 0 \\
-\pi q_2 - \frac{\partial f(g_2)}{\partial g_2} - \delta_2 &\leq 0; & \left(-\pi q_2 - \frac{\partial f(g_2)}{\partial g_2} - \delta_2 \right) g_2 &= 0 \\
y - x - \pi g_1 q_1 - \pi g_2 q_2 - f(g_1) - f(g_2) &\geq 0; & (y - x - \pi g_1 q_1 - \pi g_2 q_2 - f(g_1) - f(g_2)) \lambda &= 0 \\
g_1 - \hat{g} &\leq 0; & (g_1 - \hat{g}) \delta_1 &= 0 \\
g_2 - \hat{g} &\leq 0; & (g_2 - \hat{g}) \delta_2 &= 0 \\
q_1 \geq 0, q_2 \geq 0, g_1 \geq 0, g_2 \geq 0 && &
\end{aligned}$$

Internal solutions for all variables imply

$$\begin{aligned}
\left(\frac{\partial u}{\partial x} - \lambda \right) &= 0; \left(\frac{\partial u}{\partial q_1} - \lambda \pi g_1 \right) = 0; \left(\frac{\partial u}{\partial q_2} - \lambda \pi g_2 \right) = 0 \\
\pi q_1 + \frac{\partial f(g_1)}{\partial g_1} &= 0; \pi q_2 + \frac{\partial f(g_2)}{\partial g_2} = 0
\end{aligned}$$

Consider, however, the case where one of the demands is optimally zero, e.g., $q_2 = 0$.

Then it follows that $-\frac{\partial f(g_2)}{\partial g_2} - \delta_2 \leq 0$, so $\delta_2 > 0$. But this implies $g_2 - \hat{g} = 0$, hence

$f(g_2) = f(\hat{g}) = 0$, and spending on the second car will never be optimal. In other words, this formulation guarantees that zero optimal demand for a given car type implies that it will not be bought.

2 Setting the stage: a simple one-car model

It will be instructive to first gain intuition on the interaction between fuel efficiency and kilometer demand, conditional on car ownership decisions. In the present and next sections we therefore study conditional household choices for households owning one and two cars, respectively.

To describe the issue of demand and fuel efficiency choice in the simplest framework, we focus in this section on households that have a single car. Absence of a second car implies that q_2 equals zero. Consumer preferences are therefore defined over the composite numeraire good x and car kilometers q_1 as given by the utility function:

$$u'(x, q_1) = u(x, q_1, 0)$$

In what follows we have deleted subscripts, given there is a single car by assumption. Also note that we assume that fuel efficiency itself does not affect utility; this assumption can easily be adapted.

It will be instructive to distinguish the consumer's problem in the short and the long run. The short run problem can be thought of as choosing optimal quantities of the aggregate consumer good and the optimal demand for kilometers, assuming that the individual currently owns a car of given fuel efficiency. The long run problem allows the consumer to optimally choose the quality of the car together with optimal consumption of goods and kilometers.

Denoting the exogenously given fuel efficiency by \bar{g} , the short run problem can be formulated as

$$\underset{x, q}{\text{Max}} \quad u'(x, q) \quad \text{s.t.} \quad x + \pi \bar{g} q = y - f(\bar{g})$$

The solution yields the demand for kilometers, conditional on fuel efficiency

$$q = q(y - f(\bar{g}), \pi \bar{g}) \tag{1}$$

Indirect utility can be written as:

$$v = v'(y - f(\bar{g}), \pi \bar{g}). \tag{2}$$

In the long run the consumer optimally chooses the car with the desired fuel efficiency. We abstract from potential restrictions on available types, and assume that a

wide range of fuel efficiencies is available on the market. Hence, the long-run optimal fuel efficiency choice is the solution of:

$$\text{Max}_g \quad v = v'(y - f(g), \pi g)$$

This leads to the first-order condition

$$-\frac{\partial v}{\partial \hat{y}} \frac{\partial f}{\partial g} + \pi \frac{\partial v}{\partial p} = 0 \quad (3)$$

where $\hat{y} = y - f(g)$. Using Roy's identity, this can be reformulated as:

$$-\frac{\partial v}{\partial \hat{y}} \left(\frac{\partial f}{\partial g} + \pi q \right) = 0 \quad (4)$$

Denoting the optimal fuel efficiency choice by g^* , optimal long-run demand for kilometers is obtained by substituting g^* in the short-run demand function (1):

$$q^{LR} = q(y - f(g^*), \pi g^*)$$

Using expression (4), it follows that

$$q^{LR} = q(y - f(g^*), \pi g^*) = -\frac{1}{\pi} \frac{\partial f(g^*)}{\partial g} \quad (5)$$

Remarkably, the long run demand for kilometers is determined uniquely by information on the marginal fixed and variable costs of higher fuel efficiency; it is independent of preferences, except for the fact that the optimal choice of fuel efficiency depends on preferences (also see Rouwendal (2008)).

We are interested in the effect of a fuel price change on the optimal fuel efficiency and on the long-run demand for car kilometers. First, to derive the impact of higher fuel prices on the optimal car type chosen, apply the implicit function theorem to (4) and work out to find:

$$\frac{\partial g^*}{\partial \pi} = \frac{\pi g^* \frac{\partial q}{\partial p} + q}{[M]} \quad (6)$$

Here

$$M = -\frac{\partial^2 f}{\partial g^2} + \pi \left[\frac{\partial q}{\partial \hat{y}} \frac{\partial f}{\partial g} - \pi \frac{\partial q}{\partial p} \right] < 0 \quad (7)$$

by the second order condition for optimal fuel efficiency choice. So the effect of a fuel price increase on car fuel efficiency is positive as long as the price elasticity of demand

for car kilometers is smaller than one in absolute value. Intuitively, if demand were very elastic, higher fuel prices reduce demand so substantially that the higher fixed cost of more fuel efficiency is out-weighted by the savings on variable costs. Note that the second order condition (7) imposes nontrivial restrictions on the function $f(g)$.

Second, consider the impact of a higher fuel price on long-run demand. Differentiating $q^{LR} = q(y - f(g^*), \pi g^*)$ we have:

$$\frac{\partial q^{LR}}{\partial \pi} = \left[-\frac{\partial q}{\partial \hat{y}} \frac{\partial f(g^*)}{\partial g} + \frac{\partial q}{\partial p} \pi \right] \frac{\partial g^*}{\partial \pi} + \frac{\partial q}{\partial p} g^* \quad (7bis)$$

The last term on the right hand side is the direct effect at constant fuel efficiency: higher fuel prices reduce demand. The other components of the expression on the right hand side are indirect effects via changes in fuel efficiency. Higher fuel prices affect fuel efficiency and this has a double effect, given by the term between square brackets. One is that more fuel efficient cars have a lower price per kilometer and this raises demand (see the final term between square brackets). A second effect is that more fuel efficient cars also have a higher fixed annual ownership cost, reducing remaining income and hence demand for kilometers (first term between square brackets). The overall indirect effect raises demand, provided the effect on fixed ownership cost is not too large relative to the rebound effect.

To determine the sign of the impact of the fuel price on long-run demand, let us rewrite (7bis) as follows:

$$\frac{\partial q^{LR}}{\partial \pi} = \frac{\partial q}{\partial p} \left[\pi \frac{\partial g^*}{\partial \pi} + g^* \right] - \frac{\partial q}{\partial \hat{y}} \frac{\partial f(g^*)}{\partial g} \frac{\partial g^*}{\partial \pi} \quad (7tris)$$

This expression implies that, given the signs derived before, that a sufficient (but by no means necessary) condition for the fuel price to reduce long-run demand is that the fuel price elasticity of the demand for (inverse) fuel efficiency is less than one in absolute value:

$$\left| \frac{\partial g^*}{\partial \pi} \frac{\pi}{g^*} \right| < 1 \Rightarrow \frac{\partial q^{LR}}{\partial \pi} < 0$$

Using (6) and (7), it easily follows that whether or not the condition $\left| \frac{\partial g^*}{\partial \pi} \frac{\pi}{g^*} \right| < 1$ is satisfied depends on the specification of the fixed ownership cost function $f(g)$, more in particular on the relative size of first and second derivatives with respect to g .

Consider a simple example. Let utility be quasi-linear as follows:

$$u = x + \alpha q - .5\beta q^2$$

The parameters α and β reflect the marginal utility of the first kilometers and the rate of decline in marginal utility when more kilometers are driven, respectively. The short-run demand for kilometers, given a car of fuel efficiency \bar{g} , is given by

$$q = \frac{\alpha - \pi \bar{g}}{\beta} \quad (8)$$

Indirect utility is

$$v = y - f(\bar{g}) + \frac{1}{2\beta}(\alpha - \pi \bar{g})^2$$

The first order condition for optimal fuel efficiency (denoted g^*) is:

$$-\frac{\partial f(g^*)}{\partial g} - \pi \left(\frac{\alpha - \pi g^*}{\beta} \right) = 0 \quad (9)$$

Higher fuel efficiency is costly in terms of fixed annual costs; but it implies benefits in lower variable user costs. Solving this equation yields optimal fuel efficiency:

$$g^* = \frac{\alpha}{\pi} + \frac{\beta}{\pi^2} \frac{\partial f(g^*)}{\partial g} \quad (10)$$

The implicit function theorem implies:

$$\frac{\partial g^*}{\partial \pi} = \frac{\alpha - 2\pi g}{\left(\pi^2 - \beta \frac{\partial^2 f(.)}{\partial g^2} \right)} \quad (11)$$

Here the denominator is negative by the second order condition for optimal fuel efficiency. Simple algebra shows that the numerator is positive provided the price elasticity is less than one in absolute value. Hence, under this condition (11) implies that higher fuel prices raise fuel efficiency. Similarly, note

$$\frac{\partial g^*}{\partial \alpha} = \frac{\pi}{\left(\pi^2 - \beta \frac{\partial^2 f(.)}{\partial g^2} \right)} < 0$$

A stronger preference for driving (implying higher demand at given prices, see (8)) yields higher fuel efficiency. Finally, as in this simple example demand (8) is independent of income, we have (see 7tris):

$$\frac{\partial q^{LR}}{\partial \pi} = -\frac{1}{\beta} \left[g^* + \pi \frac{\partial g^*}{\partial \pi} \right]$$

3 Full optimum of two-car households

In this section we consider decisions by households that choose to own two cars. For such households, we study the conditional problem of optimal kilometer demand and fuel efficiency choices for the two cars. We denote the demand for kilometers by q_1, q_2 for the first and second cars, respectively; similarly, we have g_1, g_2 for the fuel efficiencies. We assume that at the optimum both $q_1 > 0, q_2 > 0$. Analogous to the one-car example studied above, we first consider optimal commodity demands for given car types (the short-run problem) and then look at the long-run problem of optimal fuel efficiency choice.

Consider the short-run problem

$$\text{Max}_{x, q_1, q_2} u(x, q_1, q_2) \text{ s.t. } x + \pi \bar{g}_1 q_1 + \pi \bar{g}_2 q_2 = y - f(\bar{g}_1) - f(\bar{g}_2)$$

where \bar{g}_1, \bar{g}_2 are given. The short-run optimization problem implies demands and indirect utilities:

$$\begin{aligned} q_1(y - f(\bar{g}_1) - f(\bar{g}_2), \bar{p}_1, \bar{p}_2) \\ q_2(y - f(\bar{g}_1) - f(\bar{g}_2), \bar{p}_1, \bar{p}_2) \end{aligned} \quad (13)$$

$$v(y - f(g_1) - f(g_2), \bar{p}_1, \bar{p}_2), \quad (14)$$

where $\bar{p}_1 = \pi \bar{g}_1; \bar{p}_2 = \pi \bar{g}_2$.

The long-run optimal inverse fuel efficiencies g_1^*, g_2^* satisfy the first order conditions of maximizing indirect utility:

$$\begin{aligned} -\frac{\partial v}{\partial \hat{y}} \frac{\partial f}{\partial g_1} + \frac{\partial v}{\partial p_1} \pi = 0 \\ -\frac{\partial v}{\partial \hat{y}} \frac{\partial f}{\partial g_2} + \frac{\partial v}{\partial p_2} \pi = 0 \end{aligned} \quad (15)$$

where $\hat{y} = y - f(g_1^*) - f(g_2^*)$. Note that, using Roy's identity, these expressions immediately imply the long run demands:

$$q_1^{LR} = q_1(y - f(g_1^*) - f(g_2^*), \pi g_1^*, \pi g_2^*) = -\frac{1}{\pi} \frac{\partial f}{\partial g_1}$$

$$q_2^{LR} = q_2(y - f(g_1^*) - f(g_2^*), \pi g_1^*, \pi g_2^*) = -\frac{1}{\pi} \frac{\partial f}{\partial g_2}$$

Again, long run demands for kilometers are uniquely determined by marginal fixed and variable cost considerations of more fuel efficient car types, at the optimal fuel efficiencies.

Now consider the impact of higher fuel prices on fuel efficiency choices of first and second cars. Reformulate the first-order conditions (15) as:

$$-\frac{\partial f}{\partial g_1} - \pi q_1(y - f(g_1) - f(g_2), \pi g_1, \pi g_2) = 0$$

$$-\frac{\partial f}{\partial g_2} - \pi q_2(y - f(g_1) - f(g_2), \pi g_1, \pi g_2) = 0$$

Differentiating and writing in matrix notation we get:

$$\begin{bmatrix} -\frac{\partial^2 f}{\partial g_1^2} - \pi \left(\frac{dq_1}{dg_1} \right) & -\pi \left(\frac{dq_1}{dg_2} \right) \\ -\pi \left(\frac{dq_2}{dg_1} \right) & -\frac{\partial^2 f}{\partial g_2^2} - \pi \left(\frac{dq_2}{dg_2} \right) \end{bmatrix} \begin{bmatrix} dg_1 \\ dg_2 \end{bmatrix} = \begin{bmatrix} \left[q_1 + \pi \left(\frac{dq_1}{d\pi} \right) \right] d\pi \\ \left[q_2 + \pi \left(\frac{dq_2}{d\pi} \right) \right] d\pi \end{bmatrix}$$

where

$$\frac{dq_i}{dg_j} = \left(\pi \frac{\partial q_i}{\partial p_j} - \frac{\partial q_i}{\partial y} \frac{\partial f}{\partial g_j} \right) \quad \frac{dq_i}{d\pi} = \left(\frac{\partial q_i}{\partial p_1} g_1 + \frac{\partial q_i}{\partial p_2} g_2 \right)$$

Solving the system by Cramer's rule yields:

$$\frac{dg_1}{d\pi} = \frac{1}{\Delta} \left\{ \left[-\frac{\partial^2 f}{\partial g_2^2} - \pi \left(\frac{dq_2}{dg_2} \right) \right] \left[q_1 + \pi \left(\frac{dq_1}{d\pi} \right) \right] + \pi \left[q_2 + \pi \left(\frac{dq_2}{d\pi} \right) \right] \left(\frac{dq_1}{dg_2} \right) \right\} \quad (16)$$

$$\frac{dg_2}{d\pi} = \frac{1}{\Delta} \left\{ \left[-\frac{\partial^2 f}{\partial g_1^2} - \pi \left(\frac{dq_1}{dg_1} \right) \right] \left[q_2 + \pi \left(\frac{dq_2}{d\pi} \right) \right] + \pi \left[q_1 + \pi \left(\frac{dq_1}{d\pi} \right) \right] \left(\frac{dq_2}{dg_1} \right) \right\} \quad (17)$$

where Δ is the determinant of the system. The second order conditions require

$$-\frac{\partial^2 f}{\partial g_i^2} - \pi \left(\frac{dq_i}{dg_i} \right) < 0 \text{ and } \Delta > 0.$$

Although one expects both signs to be negative (higher fuel prices induce people to buy more fuel efficient cars), they are ambiguous in general. However, assuming that, for a given fuel efficiency, the elasticity of travel demand with respect to the fuel price is smaller than one in absolute value, so that $\left[q_i + \pi \left(\frac{dq_i}{d\pi} \right) \right] > 0$, the expected result will hold unless cross effects of fuel efficiency are very large.

Using the same argument as in Section 2, we can again differentiate $q_i^{LR}(y - f(g_1^*) - f(g_2^*), \pi g_1^*, \pi g_2^*)$ to show that a sufficient condition for the impact of a fuel price increase on long run demand for kilometers by car i to be negative is that the elasticities of inverse fuel efficiency with respect to the fuel price are both less than one in absolute value.

Let us again consider an example. Assume

$$u = x + \alpha_1 q_1 + \alpha_2 q_2 - .5\beta_1 q_1^2 - .5\beta_2 q_2^2 - \gamma q_1 q_2.$$

where the α_i 's and β_i 's have the same interpretation as before, and the parameter γ captures the dependency of the marginal utility of extra kilometers by a given car on the kilometers driven by the other car; it is related to substitution possibilities between the two cars, see below. For given fuel efficiencies, the derived demands are given by

$$\begin{aligned} q_1 &= \frac{(\alpha_1 \beta_2 - \alpha_2 \gamma) - \beta_2 \bar{p}_1 + \gamma \bar{p}_2}{\beta_1 \beta_2 - \gamma^2}, \\ q_2 &= \frac{(\alpha_2 \beta_1 - \alpha_1 \gamma) - \beta_1 \bar{p}_2 + \gamma \bar{p}_1}{\beta_1 \beta_2 - \gamma^2}. \end{aligned} \tag{18}$$

We do, of course, expect that the intercepts of both demand equations are positive, but this is not automatically implied by the positive signs of the coefficients α_i , β_i and γ .

We will therefore assume this explicitly, so:

$$\begin{aligned} \alpha_1 \beta_2 - \alpha_2 \gamma &> 0 \\ \alpha_2 \beta_1 - \alpha_1 \gamma &> 0 \end{aligned} \tag{19}$$

We furthermore assume that the second order condition is fulfilled. This implies:

$$\beta_1\beta_2 - \gamma^2 > 0 \quad (20)$$

Note that all three conditions state – in different ways – that the parameter γ should not be ‘too large’ for the model to make sense. One may interpret this as saying that two cars will only be used if substitution is not too easy.

Indirect utility can be expressed as:

$$v(y - f(\bar{g}_1) - f(\bar{g}_2), \bar{p}_1, p_2) = y - f(\bar{g}_1) - f(\bar{g}_2) + \frac{1}{2} \frac{\beta_2(\alpha_1 - \bar{p}_1)^2 + \beta_1(\alpha_2 - \bar{p}_2)^2 - 2\gamma(\alpha_1 - \bar{p}_1)(\alpha_2 - \bar{p}_2)}{\beta_1\beta_2 - \gamma^2}$$

Optimal long run fuel efficiency choices (g_1^*, g_2^*) are the solution to the conditions

$$\begin{aligned} \frac{\partial v}{\partial g_1} &= -\frac{\partial f}{\partial g_1} - \left(\frac{\pi}{\beta_1\beta_2 - \gamma^2} \right) [\beta_2(\alpha_1 - \pi g_1) - \gamma(\alpha_2 - \pi g_2)] = 0 \\ \frac{\partial v}{\partial g_2} &= -\frac{\partial f}{\partial g_2} - \left(\frac{\pi}{\beta_1\beta_2 - \gamma^2} \right) [\beta_1(\alpha_2 - \pi g_2) - \gamma(\alpha_1 - \pi g_1)] = 0 \end{aligned} \quad (21)$$

Solving for optimal fuel efficiency choices implies:

$$\begin{aligned} g_1^* &= \frac{\alpha_1}{\pi} + \frac{1}{\pi^2} \left(\beta_1 \frac{\partial f(g_1^*)}{\partial g_1} + \gamma \frac{\partial f(g_2^*)}{\partial g_2} \right) \\ g_2^* &= \frac{\alpha_2}{\pi} + \frac{1}{\pi^2} \left(\beta_2 \frac{\partial f(g_2^*)}{\partial g_2} + \gamma \frac{\partial f(g_1^*)}{\partial g_1} \right) \end{aligned} \quad (22)$$

Long run demands are

$$\begin{aligned} q_1^{LR} &= -\frac{1}{\pi} \frac{\partial f(g_1^*)}{\partial g_1} \\ q_2^{LR} &= -\frac{1}{\pi} \frac{\partial f(g_2^*)}{\partial g_2} \end{aligned}$$

To determine the effect of higher fuel prices on fuel efficiency choices for the two cars, differentiate the first order conditions (21) to find:

$$\begin{aligned} \left(-\frac{\partial^2 f}{\partial g_1^2} + \frac{\beta_2 \pi^2}{Z} \right) dg_1 - \left(\frac{\gamma \pi^2}{Z} \right) dg_2 - \left\{ \frac{1}{Z} [(\beta_2 \alpha_1 - \gamma \alpha_2) + 2\pi(\gamma g_2 - \beta_2 g_1)] \right\} d\pi &= 0 \\ \left(-\frac{\partial^2 f}{\partial g_2^2} + \frac{\beta_1 \pi^2}{Z} \right) dg_2 - \left(\frac{\gamma \pi^2}{Z} \right) dg_1 - \left\{ \frac{1}{Z} [(\beta_1 \alpha_2 - \gamma \alpha_1) + 2\pi(\gamma g_1 - \beta_1 g_2)] \right\} d\pi &= 0 \end{aligned}$$

where $Z = \beta_1\beta_2 - \gamma^2 > 0$. Working out yields:

$$\frac{dg_1}{d\pi} = \left(\frac{1}{\Delta}\right)\left(\frac{1}{Z}\right) \left\{ \pi^2 (\alpha_1 - 2\pi g_1) - \frac{\partial^2 f}{\partial g_2^2} [(\beta_2 \alpha_1 - \gamma \alpha_2) + 2\pi(\gamma g_2 - \beta_2 g_1)] \right\}$$

$$\frac{dg_2}{d\pi} = \left(\frac{1}{\Delta}\right)\left(\frac{1}{Z}\right) \left\{ \pi^2 (\alpha_2 - 2\pi g_2) - \frac{\partial^2 f}{\partial g_1^2} [(\beta_1 \alpha_2 - \gamma \alpha_1) + 2\pi(\gamma g_1 - \beta_1 g_2)] \right\}$$

where $\Delta > 0$ (by the second order conditions) is the determinant of the system. The signs of the fuel price effects are ambiguous in general; if kilometer demand is inelastic (implying $(\alpha_i - 2\pi g_i) > 0$) and γ is not too large, then both effects are negative.

Finally, it is easily shown that we have unambiguously $\frac{dg_i}{d\alpha_i} < 0$. This implies that a stronger preference for driving car i results in the selection of a more fuel efficient car i .

4. Optimal fuel efficiency choice of a second car, conditional upon the first

Most households buy their first and second cars sequentially. To capture this phenomenon, consider a household that has a single car of given fuel efficiency \bar{g}_1 (which may or may not be optimal in the long run), and it decides to buy a second car. This implies that the household's current utility with two cars exceeds that of having one car (for more details, see below). The household can optimally select the fuel efficiency for the second car to be bought, and it can decide on the kilometers driven by both cars, conditional on having the first car available.

The 'short-run' problem is the same as above. Short-run demands and indirect utility are the same as before. Optimal fuel efficiency choice for the second car leads to the first-order condition, using Roy:

$$-\frac{\partial v}{\partial \hat{y}} \left(\frac{\partial f(g_2)}{\partial g_2} + \pi q_2 \right) = 0$$

Importantly, note that demand for kilometers is $q_i(y - f(\bar{g}_1) - f(g_2), \pi \bar{g}_1, \pi g_2)$, reflecting the existing car's fuel efficiency.

To find the impact of higher fuel prices and of a more fuel efficient first car available on the optimal car type chosen, apply the implicit function theorem to find:

$$\frac{\partial g_2^*}{\partial \pi} = \frac{\pi g_2 \frac{\partial q_2}{\partial p_2} + q_2}{[L]}, \quad L = -\frac{\partial f^2}{\partial g_2^2} + \pi \left[\frac{\partial q_2}{\partial \hat{y}} \frac{\partial f}{\partial g_2} - \pi \frac{\partial q_2}{\partial p_2} \right]$$

$$\frac{\partial g_2^*}{\partial \bar{g}_1} = -\frac{\pi^2 \frac{\partial q_2}{\partial p_1}}{[L]} < 0$$

Note that L is negative by the second order condition for optimal fuel efficiency choice.

The effect of a fuel price increase on car fuel efficiency is positive as long as the price elasticity of demand for car kilometers is smaller than one in absolute value. Interestingly, having a more fuel efficient car available reduces the fuel efficiency of the new car bought, provided cross-price effects are positive. A more fuel efficient first car implies, at given fuel prices, a lower price per kilometer for use of the first car. Positive cross price effects imply then lower demand for use of the second car, reducing the benefits of a more fuel efficient second car.

Consider again the example given above. The derived demands are now given by

$$q_1 = \frac{(\alpha_1 \beta_2 - \alpha_2 \gamma) - \beta_2 \bar{p}_1 + \gamma p_2}{\beta_1 \beta_2 - \gamma^2},$$

$$q_2 = \frac{(\alpha_2 \beta_1 - \alpha_1 \gamma) - \beta_1 p_2 + \gamma \bar{p}_1}{\beta_1 \beta_2 - \gamma^2}. \quad (23)$$

where

$$\bar{p}_1 = \pi \bar{g}_1$$

$$p_2 = \pi g_2$$

This notation reflects the fact that the first, but not the second, car's fuel efficiency is given. Substituting yields, after rearrangement, indirect utility:

$$v(y - f(\bar{g}_1) - f(g_2), \bar{p}_1, p_2) = y - f(\bar{g}_1) - f(g_2) + \frac{1}{2} \frac{\beta_2 (\alpha_1 - \bar{p}_1)^2 + \beta_1 (\alpha_2 - p_2)^2 - 2\gamma (\alpha_1 - \bar{p}_1) (\alpha_2 - p_2)}{\beta_1 \beta_2 - \gamma^2}$$

The condition for optimal choice of fuel efficiency of the second car is given by:

$$\frac{\partial v}{\partial g_2} = -\frac{\partial f}{\partial g_2} - \pi \left[\frac{(\alpha_2 \beta_1 - \alpha_1 \gamma) - \beta_1 p_2 + \gamma \bar{p}_1}{\beta_1 \beta_2 - \gamma^2} \right] = 0$$

This just reflects, see Roy's identity: $q_2 = -\frac{1}{\pi} \frac{\partial f}{\partial g_2}$. Noting that $p_2 = \pi g_2$ the solution for the optimal fuel efficiency of the second car yields:

$$g_2^* = \left[\frac{(\beta_1 \beta_2 - \gamma^2) \frac{\partial f}{\partial g_2} + \pi [\beta_1 \alpha_2 - \gamma(\alpha_1 - \bar{p}_1)]}{\beta_1 \pi^2} \right] \quad (24)$$

Differentiating the above first-order condition, taking account of the fact that this is not an explicit solution due to the dependency of $\frac{\partial f}{\partial g_2}$ on g_2 , we have:

$$\frac{\partial g_2^*}{\partial \bar{g}_1} = \left[\frac{\gamma \pi^2}{(\beta_1 \beta_2 - \gamma^2)(N)} \right] < 0$$

$$\frac{\partial g_2^*}{\partial \pi} = \left[\frac{(\alpha_2 \beta_1 - \gamma \alpha_1) - 2\pi(\beta_1 g_2 - \gamma g_1)}{(\beta_1 \beta_2 - \gamma^2)(N)} \right]$$

where, by the second order conditions, $N < 0$. It follows that having a more fuel efficient car reduces the fuel efficiency of the second car; moreover, higher fuel prices plausibly raise the fuel efficiency of the second car. These are empirically testable implications.

Use of the two cars is given by, after substituting the optimal fuel efficiency for the second car in the demand functions:

$$q_1^* = \frac{(\alpha_1 - \pi \bar{g}_1)}{\beta_1} + \frac{\gamma}{\beta_1 \pi} \frac{\partial f}{\partial g_2} \quad (25)$$

$$q_2^* = -\frac{1}{\pi} \frac{\partial f}{\partial g_2} \quad (26)$$

The second car is 'optimally used'; the optimal demand for kilometers is independent of the fuel efficiency of the first car, and it equals the long run optimal demand we had before for the case both car fuel efficiencies could be optimally chosen (see (15) and the discussion below that equation). The first car is used more (i) the higher the intrinsic utility of driving it, (ii) the higher its given fuel efficiency and (iii) the less substitution possibilities there exist with the second car.

What happens to kilometer demand due to having the second car? Does total demand rise? Will the second car be more fuel efficient than the first car bought? Will the second car be used more or less intensively than the first one?

Let us try to answer these questions. First, note that in the case the household only had one car, demand was

$$q_1^*(1) = \frac{\alpha_1 - \pi \bar{g}_1}{\beta_1}$$

This implies having the second car reduces use of the first car (compare with (25)) provided there is some substitution possibility. Total car demand by the two cars amounts to

$$q_1^* + q_2^* = \frac{(\alpha_1 - \pi \bar{g}_1)}{\beta_1} + \frac{1}{\pi} \frac{\partial f}{\partial g_2} \left(\frac{\gamma - \beta_1}{\beta_1} \right) = q_1^*(1) + \frac{1}{\pi} \frac{\partial f}{\partial g_2} \left(\frac{\gamma - \beta_1}{\beta_1} \right)$$

Provided there is not too much substitution, total demand rises, because then the final term on the right hand side is positive.

Second, will the second car be more fuel efficient than the first? The second car will be more fuel efficient than the first one if, see (24):

$$\bar{g}_1 < \left[\frac{(\beta_1 \beta_2 - \gamma^2) \frac{\partial f(g_2^*)}{\partial g_2} + \pi [\beta_1 \alpha_2 - \gamma (\alpha_1 - \bar{p}_1)]}{\beta_1 \pi^2} \right]$$

Noting that $\bar{p}_1 = \pi \bar{g}_1$ and working out yields:

$$\bar{g}_1 < \left[\frac{(\beta_1 \beta_2 - \gamma^2) \frac{\partial f(g_2^*)}{\partial g_2} + \pi [\beta_1 \alpha_2 - \gamma \alpha_1]}{\pi^2 (\beta_1 - \gamma)} \right]$$

Third, which car will be used most intensively? The first car will be used more than the second ($q_1^* > q_2^*$) if its inverse fuel efficiency satisfies (see (25)-(26)):

$$\bar{g}_1 < \frac{\alpha_1}{\pi} + \frac{(\gamma + \beta_1)}{\pi^2} \frac{\partial f(g_2^*)}{\partial g_2}$$

5. The impact of higher fuel prices on car ownership, fuel efficiency, travel demand and total fuel consumption.

What are the effects of fuel price increases on travel demand and fuel use in the long-run, knowing that people select optimally the number of cars they want to own and the fuel efficiency of these cars? To answer these questions, we have to model car ownership decisions; conditional decisions were studied before.

We now have to make notation more precise to study the discrete choices of car ownership. We denote the optimal demands for use of the j 'th car by people owning i cars by q_j^i . Similarly, the inverse fuel efficiency of the j 'th car by people owning i cars is g_j^i . We can then define the unconditional utilities of households having zero, one or two cars (also formulated in section 1) alternatively as follows:

$$\begin{aligned} w^0(y) \\ w^1(\pi, y) &= \underset{g_1^1}{\text{Max}} v^1(y - f(g_1^1), \pi g_1^1) \\ w^2(\pi, y) &= \underset{g_1^2, g_2^2}{\text{Max}} v^2(y - f(g_1^2) - f(g_2^2), \pi g_1^2, \pi g_2^2) \end{aligned}$$

Note that the indirect utilities w^i are conditional on how many cars are owned, but they do assume optimal fuel efficiency choice, conditional on ownership. Observe that Roy's identity implies:

$$\begin{aligned} \frac{\partial w^1}{\partial \pi} &= -q_1^1 \frac{\partial w^1}{\partial y} g_1^1 \\ \frac{\partial w^2}{\partial \pi} &= -q_1^2 \frac{\partial w^2}{\partial y} g_1^2 - q_2^2 \frac{\partial w^2}{\partial y} g_2^2 \end{aligned}$$

If one assumes, for simplicity, constant marginal utility of income equal to one, then we have:

$$\begin{aligned} \frac{\partial w^1}{\partial \pi} &= -q_1^1 g_1^1 \\ \frac{\partial w^2}{\partial \pi} &= -q_1^2 g_1^2 - q_2^2 g_2^2 \end{aligned}$$

There are two obvious ways to model the discrete choice of car ownership. One is to assume people have different preferences (and/or incomes), but that the distribution of

the relevant parameters is known. For example, if -- conditional on income -- the distribution of preference parameters is known this generates the probabilities of owing zero, one or two cars as function of income and the fuel price. Another approach (see De Borger and Mayeres (2007)) is to assume that people have identical ‘observable’ preferences but that there are unobservable differences in tastes that result in different choices. Assuming a joint distribution for these unobservables then results in the probabilities that households have zero, one or two cars².

We will follow the first approach in the numerical simulation exercise of the next section. Let us here just denote the fraction of households choosing zero, one or two cars by r_i ($i=0,1,2$). Long run total car transport demand Q and total fuel demand F are then given by:

$$Q = r_1 q_1^1 + r_2 (q_1^2 + q_2^2)$$

$$F = r_1 q_1^1 g_1^1 + r_2 (q_1^2 g_1^2 + q_2^2 g_2^2)$$

Here all demands are long-run demands, i.e., assuming optimal fuel efficiency choices. So we can formulate the impact of higher fuel prices on demand and fuel use as, respectively:

$$\frac{\partial Q}{\partial \pi} = r_1 \left[\frac{\partial q_1^1}{\partial \pi} \right] + r_2 \left[\frac{\partial q_1^2}{\partial \pi} + \frac{\partial q_2^2}{\partial \pi} \right] + q_1^1 \frac{\partial r_1}{\partial \pi} + (q_1^2 + q_2^2) \frac{\partial r_2}{\partial \pi}$$

² Assume Weibull distributions for the unobservables, then the probability r_i of owning i

cars can be written: $r_i = \frac{e^{w^i}}{\sum_{j=0}^2 e^{w^j}}$. Simple algebra then shows:

$$\frac{\partial r_0}{\partial \pi} = r_0 \left\{ r_1 q_1^1 g_1^1 + r_2 [q_1^2 g_1^2 + q_2^2 g_2^2] \right\}, \frac{\partial r_1}{\partial \pi} = r_1 \left\{ r_2 [q_1^2 g_1^2 + q_2^2 g_2^2] - (1 - r_1) q_1^1 g_1^1 \right\}$$

and $\frac{\partial r_2}{\partial \pi} = r_2 \left\{ r_1 q_1^1 g_1^1 - (1 - r_2) [q_1^2 g_1^2 + q_2^2 g_2^2] \right\}$.

Note that this implies that higher fuel prices raise the probability of not having a car, but the impact on the probabilities of having one or two cars is ambiguous. If there are few two-car households and their kilometer demand is substantially larger than for one-car households, then the impact of higher fuel prices on the probability of having two cars declines. If the share of households having a single car is not too large then, at the same time, the probability of having a single car rises.

$$\begin{aligned} \frac{\partial F}{\partial \pi} = & r_1 g_1^1 \frac{\partial q_1^1}{\partial \pi} + r_1 q_1^1 \frac{\partial g_1^1}{\partial \pi} + r_2 \left[g_1^2 \frac{\partial q_1^2}{\partial \pi} + g_2^2 \frac{\partial q_2^2}{\partial \pi} \right] + r_2 \left(q_1^2 \frac{\partial g_1^2}{\partial \pi} + q_2^2 \frac{\partial g_2^2}{\partial \pi} \right) \\ & + q_1^1 g_1^1 \frac{\partial r_1}{\partial \pi} + (q_1^2 g_1^2 + q_2^2 g_2^2) \frac{\partial r_2}{\partial \pi} \end{aligned}$$

Higher fuel prices reduce long-run demands at given ownership probabilities, and they affect total demand via changes in the ownership probabilities; the latter effects can be positive or negative. One expects, however, total effects to be negative. The impact on total fuel demand has the additional effect of fuel prices on fuel efficiency.

6. Numerical illustration

Specifications

In this section we provide a numerical illustration of some implications of the model. To start, we repeat eq. (5), which gives an expression for the long run demand function of a single car household:

$$q^{LR} = -\frac{1}{\pi} \frac{\partial f(g^*)}{\partial g}$$

This relationship also holds when there are two cars in the household (see below eq. 22). This equation implies that there is a close relationship between the number of kilometers driven, the fuel price and the curvature of the fixed monthly ownership curve. For a given value of the fuel price, there is a one-to-one relationship between the demand for kilometers and the slope of the fixed cost curve. Large demand implies high fuel efficiency (a low value of g), low demand a low level of fuel efficiency. The variation in demand is also closely related to the variation in fuel efficiency. Currently the most efficient cars use a little more than 4 liters of gasoline per 100 km driven. Most cars are much less fuel efficient. The average value is probably close to 7, and although there is not a minimum fuel efficiency, it appears exceptional that cars use more than 11 liters per 100 km driven. The range of kilometers driven per month reaches from almost 0 to more than 10.0000. The average for single car households is approximately 1000 km per month. Assuming (mainly for convenience) a fuel price of 1 would therefore imply that

the slope of the fixed cost function would have to decrease rapidly between $g=4$ and $g=11$.

It is also important to observe at this point that our model implies that the effect of fuel price changes on fuel efficiency are similar to those of a higher demand (at a given fuel price). Indeed, multiplication of both sides of (5) by π makes clear that monthly fuel expenditure and the choice of fuel efficiency are closely related.

For the numerical simulations that follow we selected the following quadratic specification of monthly fixed cost:

$$f(g_i) = \delta_0 + \delta_1 g_i + \delta_2 (g_i)^2$$

We assume, as before in our illustrations, that the utility function is given as:

$$u = x + \alpha_1 q_1 + \alpha_2 q_2 - .5\beta_1 q_1^2 - .5\beta_2 q_2^2 - \gamma q_1 q_2.$$

where the parameters were interpreted above. The demand functions associated with this utility function in case two cars are owned have been derived above, see (18). If only one car is owned, it is always car 1, and for this case demand has been derived in (8).

Optimal choice of fuel efficiency follows from (5). In case only one car is owned, we can derive the following expression for g_1 :

$$g_1 = \frac{\alpha_1 \pi + \beta \delta_1}{\pi^2 - 2\beta \delta_2}.$$

When two cars are owned, the expressions for optimal fuel efficiency are more complicated:

$$g_1 = \frac{\pi^2 (\pi \alpha_1 + (\gamma + \beta_1) \delta_1) + 2 (\pi \alpha_2 \gamma - \pi \alpha_1 \beta_2 - (\beta_1 \beta_2 - \gamma^2) \delta_1) \delta_2}{\pi^4 - 2\pi^2 (\beta_1 + \beta_2) \delta_2 + 4 (\beta_1 \beta_2 - \gamma^2) \delta_2^2}$$

$$g_2 = \frac{\pi^2 (\pi \alpha_2 + (\gamma + \beta_2) \delta_1) + 2 (\pi \alpha_1 \gamma - \pi \alpha_2 \beta_1 - (\beta_1 \beta_2 - \gamma^2) \delta_1) \delta_2}{\pi^4 - 2\pi^2 (\beta_1 + \beta_2) \delta_2 + 4 (\beta_1 \beta_2 - \gamma^2) \delta_2^2}$$

It is possible to substitute the expressions for the optimal fuel efficiency levels back in the indirect utility functions referring to ownership of one or two cars. This leads to analytical solutions that can be used to solve (also analytically) for the critical values of α_1 and α_2 at which a household is indifferent between owning 0 and 1 car, or between

owning 1 and 2 cars. However, these expressions are too complicated to be easily interpretable and for that reason we will instead make use of simulations.

Parameter values

The parameters of the fixed cost function were chosen in such a way that:

- the parabola reaches a minimum for $g=11$,
- monthly fixed costs are equal to 250 euro for $g=7$, and
- at $g=7$ and $\pi=1$ the number of km per month equals 1000.

This leads to the following parameter values:

$$\begin{array}{ll} \delta_2 & 1,250 \\ \delta_1 & -27,50 \\ \delta_0 & 381,25 \end{array}$$

The minimum fixed cost is reached at $g=11$ and equals 230 euro per month. In the model consumers will never choose a higher value of g . The quadratic specification implies that there is not a minimum value of g . Although the three requirements used to derive these parameters are certainly useful, they do not imply that the resulting cost function is convincing in all aspects. One concern is that the sensitivity (first derivative) of f with respect to g seems small. For example, raising g from 4 to 5 only implies a fixed cost reduction of 17.5 euro per month, which seems to be on the low side.

Since we express fuel efficiency in the number of liters per 100 km, we also express demand in units of 100 km. The following base case parameters were used:

$$\begin{array}{ll} \beta_1 = \beta_2 & 7 \\ \gamma & 1 \end{array}$$

The parameters α_1 and α_2 differ over the households. At the base values of the parameters, the minimum value of α_1 at which the household decides to own a car equals 67. At this critical value the household drives almost 850 km per month and has an optimal fuel use per 100 km that equals 7.6. Demand is equal to 1000 km per month when α_1 is equal to 77, and then the price elasticity is -0.1, while the optimal car uses 7 liters per 100 km. When α_1 equals 100 almost 1350 km per month are driven and optimal fuel efficiency is 5.6 liters per 100 km.

The consumer surplus implied by the demand equation for a single car household equals 350 euro per month when 1000 km are driven. This means that fixed costs have to

be less than this amount if we want this household to own a car. That is the reason why we set the value of the fixed cost at $g=7$ at 250, although this seems somewhat low.

We assume that the value of α_2 is at most equal to that of α_1 , which implies that car 1 is always the first car: it is the car with the highest monthly number of kilometers and if a household owns one car, it is always car 1. The distribution of the α s is uniform:

$$f(\alpha_1, \alpha_2) = c \quad \text{if } \alpha_1, \alpha_2 \geq \alpha^{\min}, \text{ and } \alpha_2 \leq \alpha_1 \leq \alpha^{\max}.$$

In this equation c is a constant that equals $1 / (0.5(\alpha^{\max} - \alpha^{\min})^2)$. In the simulations we use $\alpha^{\min}=50$ and $\alpha^{\max}=100$.

Effects of changes in fuel prices when households can have 1 car

We start with a simplified version of the model in which households can only have one car, no matter what their value of α_2 is. We used the marginal distribution of α_1 implied by the distribution f . introduced at the end of the previous section.

In situation 0 the fuel price is equal to 1.

Table 1 Effects of an increase in the fuel price: single car model

	Before the change	Short run adjustment	Long run adjustment
Π	1.00	1.25	1.25
Share of car owners	89.6%	89.6%	87.2%
Average monthly km	1136	1113	1141
Av. Fuel eff. (liters per 100 km)	6.45	6.45	5.29
Max adj in km	-	-27	-13
Min adj in km	-	-20	4

The average change in km for households owning a car before and after the price change is negative (although small): -3 km. The increase in the average number of monthly km is therefore completely caused by the small number over km driven by those who abandoned their car (849).

The most surprising aspect of Table 1 is that for some households the long run effect of an increase of the fuel price on the number of kilometers driven is positive. Although this is unexpected, it is consistent with the model. Indeed, it follows from eq. 7 that the long run demand function is upward sloping if the elasticity of inverse fuel efficiency with respect to the fuel price is larger than one in absolute value and there are no income effects. The latter condition is satisfied in the simulation model used here, which is based on a quasi-linear utility function. We regard this as a curiosity, rather than as a significant result.

Table 2 presents simulation results for the complete model in which households can own either one or two cars. Although the average number of kilometers of car owner households decreases somewhat, the average number of kilometers driven by single-car and two-car households is higher after the long run adjustment than it was before the price change. Although the relatively large effect of the fuel price increase on fuel efficiency helps to generate this effect, switches in the number of cars are the decisive factor. As a result of the higher fuel price, 2.4% of the household abandon their single car, while 3.3% switch from 2 to 1 cars. These households all realize large negative effects in the total number of kilometers driven (see the last two lines of the table), but the households that switch from two cars to one are those with a relatively low number of kilometers in the group of two car households, and they become one car households with a relatively high number of kilometers. Similarly, the households that decide to abandon their single car are those with a relatively small demand for kilometers.

Another remarkable aspect of Table 2 is that the average fuel efficiency in the total car fleet decreases after adjustments to the higher fuel prices have been realized. Again, this is caused by a composition effect: the table shows that households that did not change their number of cars realized a substantial increase in fuel efficiency.

Table 2 Effects of an increase in the fuel price: multiple car model

	Before the change	Short run adjustment	Long run adjustment
Fuel price	1.00	1.25	1.25
Share of households without a car	10.4%	10.4%	12.8%
Share of households with one car	68.7%	68.7%	69.6%
Share of households with two cars	20.9%	20.9%	17.6%
Average monthly km all car owning households	1352	1324	1326
- Households with 1 car	1109	1086	1117
- Households with two cars	2149	2106	2153
- - Car 1	1079	1058	1083
- - Car 2	1070	1047	1070
Av. Fuel eff. (liters per 100 km)	6.45	6.45	6.68
- Households with one car	7.51	7.51	7.08
- Car 1 of households with two cars	6.68	6.68	5.59
- Car 2 of households with two cars	7.19	7.19	6.18
Av. Change in km all car owning households		-28	-63
- Households with one car		-23	-4*
- Households with two cars		-43	-12*
- Households abandoning their single car			-849
- Households switching from 2 to 1 car			-900

* This figure refer to households with the same number of cars before and after the change.

Figure 1 illustrates the effects of a fuel price increase on multiple car ownership by means of a diagram that has the α_1 on the horizontal axis and α_2 on the vertical one. The relevant combinations of these two parameters are those for which $\alpha_1 < \alpha_2$, and the boundary of this area is indicated by the diagonal line. The other three lines indicate the combinations of α s at which households decide to have 0,1 or 2 cars. To the left of the

lines households do not own a car, above the lines households own two cars and below and to the right of these lines households own one car. When the fuel price increases, some households shift from two cars to one, others from one car to none. The share of households with two cars decreases, the share of households without a car increases, but the effect on the number of households owning no car at all is ambiguous.

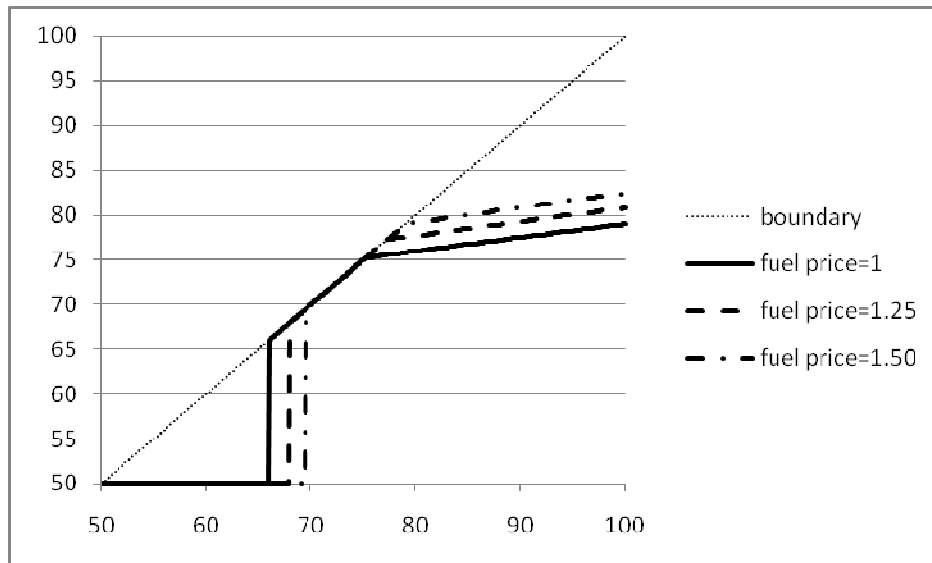


Figure 1 Effects of the fuel price on car ownership

7 Conclusion

In the previous sections we developed a model that allows us to study multiple car ownership, fuel efficiency and kilometers driven in a consistent microeconomic framework. Special cases of the model refer to the short-run problem of determining the optimal number of kilometers taking fuel efficiency as given, or the medium-run problem of determining the fuel efficiency of a second car to be bought, while taking the characteristics of the first car as given. We studied the cases in which the number of cars is predetermined at 1 or 2 separately, and then moved to the general discrete-continuous choice model in which all three aspects can be studied simultaneously.

We developed a numerical version of the model, and used it to investigate the effects of changes in fuel prices and substitution possibilities on car ownership, fuel efficiency and number of kilometers driven. A potentially important suggestion of these

exercises is that changes in the composition of the population of households with 0, 1 or 2 cars lead to the surprising result that increases in fuel prices lead to an increase in the average number of kilometers driven by both single car and two-car households. Individual households do nevertheless adjust the number of kilometers driven in the expected (downward) direction although this effect is mitigated by adjustments in fuel efficiency. The average number of cars owned and the average number of kilometers driven in the population both decrease as a consequence of the higher fuel price.

A second potentially important implication is that the possibilities to substitute kilometers driven between two cars has a substantial effect on the decision to use a second car. In our model this substitution effect is present in the demand functions for kilometers driven by each car: if the price of using one car increases, the number of kilometers driven by this car decreases whereas the number of kilometers driven by the other car increases.

The results of the paper should be regarded as preliminary. We plan to do further work on the effect of taxes on the demand for kilometers, fuel use and car ownership and to consider the possible attractiveness of a separate tax treatment of second cars.

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