A Representative Consumer Framework
for Discrete Choice Models with Endogenous Total Demand

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Abstract: Standard discrete choice models correspond to 'partial' utility maximization in which the total demand is determined exogenously; typically, consumers are assumed to demand at most one unit. The purpose of this paper is to formulate a model in which discrete choice models are incorporated consistently into the full utility maximization framework and to establish a theoretical foundation for discrete choice models that assume no a priori total demand. We derive the form of the corresponding indirect utility function of a representative consumer and the own-price and cross-price elasticities, and develop a method for measuring welfare, clarifying their implications.

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1. Introduction

Discrete choice models are suitable for representing a consumer's micro behavior. For example, consider a problem in which a consumer chooses between one unit of brands A and B of a particular good. Comparing the utility levels obtained from brands A and B, the consumer chooses the brand that yields higher utility. This setup is consistent with utility maximization by the consumer. However, this problem represents only part of the consumer's behavior. This is because the reason for the consumer choosing a unit of the good is not explained. The outside option of 'not buying', which involves not purchasing either brand, does not completely solve the problem because, in this case, one still needs to make an a priori assumption that the consumer's demand from a choice set (brand A, brand B, and the outside option in this example) is restricted to one[2]. Thus, typical discrete choice models explain only part of consumer behavior because their use involves making an a priori assumption about the total demand for a choice set, which is exogenous. This has been repeatedly argued by McFadden (1999, p. 273), Nevo (2000, footnote 14), and Nevo (2001, footnote 13), but no clear theoretical foundation for the argument has been developed.[3]

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[2] Another solution is to construct a discrete/continuous model as in Dubin and McFadden (1984), Hanemann (1984), and Hendel (1999). However, these analyses preclude the possibility that a consumer or a firm chooses multiple brands for the single purpose.

[3] Nevo (2001, footnote 13) states: "A comment is in place about the realism of the assumption that consumers choose no more than one brand. Many households buy more than one brand of cereal in each super market trip but most people consume only one brand of cereal at a time, which is the relevant fact for this modeling assumption. Nevertheless, if one is still unwilling to accept that this is a negligible phenomenon, then this model can be viewed as an approximation to the true choice model." Nevo (2000, footnote 14) makes the same point. McFadden (1999, p. 273) raises the possibility that an alternative can be interpreted "as a 'portfolio' of decisions made in sequence, or as one of the multiple decisions."

The purpose of this paper is to formulate a model in which discrete choice models are incorporated consistently into the full utility maximization framework and to establish a theoretical foundation for the discrete choice model that assumes no a priori total demand for a choice set. In our framework, the results from discrete choice models are explained consistently whether a total demand for a choice set is exogenous or endogenous: the case of an exogenous total demand is a special case. The implications of discrete choice models are also clarified, because the form of the utility function of the representative consumer, the own-price and cross-price elasticities, and the method of measuring welfare are derived in such a way that our results are directly comparable with standard microeconomic utility maximization.

We focus on the generalized extreme value (GEV) model, and its mixed form, because it generates analytically closed-form demand functions. Given that Dagsvik (1995) and McFadden and Train (2000) show that the GEV model and the mixed logit model can approximate any random utility model, our analysis is quite general. For illustrative purposes, we focus on the logit and mixed logit models as special cases before fully analyzing the GEV and mixed GEV models. As we show subsequently, analyzing the GEV and mixed GEV models is similar to analyzing the logit and mixed logit models.

Our main results are as follows. First, if a representative consumer's choice is represented by the logit model, his or her demand function for a good has the form of the market demand function for a group of goods multiplied by the choice probability of the good. (The market demand function for a group of goods is the sum of the market demands across all goods.) This form of the demand function is obtained when the indirect utility function of the representative consumer follows Gorman's (1961) framework and incorporates its restriction, and when the log-sum term is incorporated as the price index. Making the market demand for a group endogenous affects the own-price and cross-price elasticities of the market demand for a good. The price elasticity of the market demand for a group of goods is added to the usual own-price and cross-price elasticities in 'classical' logit models. Throughout the paper, we use the word 'classical' to represent a situation in which a consumer is assumed to make a single selection among a set of mutually exclusive alternatives. The change in welfare can be measured by using any of the four levels of demand: a consumer's demand for a good, the market demand for a good, a consumer's demand for a group of goods, or the market demand for a group of goods. If we measure the change in welfare by using a consumer's demand for a good or the market demand for a good, the corresponding price is the price of the good. If we measure the welfare change by using a consumer's demand for a group of goods or the market demand for a group of goods, the corresponding price is the log-sum term. The change in welfare in the classical logit model is typically calculated as the difference in the log-sum term multiplied by the total demand; this method is a special case of our analysis.

Second, our analysis can easily be extended to cases in which goods are classified into multiple groups. In this case, we can construct a model in which the choice within each group, such as between food and clothes, is represented by the logit model but the choice among groups is subject to any relationship. Not restricting relationships between groups is an advantage of our formulation. For example, the nested-logit model incorporates the grouping of goods, but the relationship between groups is limited to the logit.

Third, analyzing the mixed logit model requires modification. In the mixed logit model, each consumer has his or her own parameters. This implies that the log-sum term, which represents his or her price index, differs among consumers. Hence, the indirect utility function of the representative consumer must be quasi-linear because all consumers must have the same income coefficient for their Gorman-form indirect utility function. Similar results are obtained with regard to elasticities: the price elasticity of the market demand for a group is added to the usual own-price and cross-price elasticities. The change in welfare can be measured by using a consumer's demand for a good or a consumer's demand for a group. A difference from the logit model is that one cannot calculate the change in welfare by using the market demand for a good or the market demand for a group. The reason for this is that because each consumer's demand depends on his or her own parameters, the market demand cannot be derived without integrating out these terms. As in the case of the logit model, our analysis can easily be extended to the case in which goods are classified into multiple groups and to the case of the mixed GEV model.

Next, we briefly relate our analysis to the existing literature. The first line of research related to our paper is analysis of the relationship between discrete choice models and representative consumer models. Anderson et al. (1988, 1992, Ch. 3) and Verboven (1996) derive direct utility functions for the representative consumer that are consistent with the logit and nested logit models. Not only does our analysis correspond to the more general GEV model, which includes their models as special cases, but also we derive a utility maximization problem that is consistent with the mixed-GEV model. Moreover, we formulate a utility maximization problem that corresponds to the GEV and mixed-GEV models in a more realistic and more general framework: goods are classified into multiple groups, and a consumer can choose any number of goods from any groups.

The second line of research is the analysis of welfare measurement for discrete choice models. First, welfare measurement for discrete choice models is theoretically analyzed by, among others, Small and Rosen (1981) in a general form, but the market demand for a group is assumed exogenous.[4] Our analysis is an extension of theirs because we make the market demand for a group endogenous and develop a method of measuring welfare that is applicable to the case of endogenous demand. Second, Herriges and Kling (1999), McFadden (1999), de

[4] See de Jong et al. (2005) for a review of the literature on welfare measurement in discrete choice models.

Palma and Kilani (2003), and Dagsvik and Karlstrom (2005) analyze welfare measurement for discrete choice models in which an indirect utility function is nonlinear in income.[5] In these analyses, the focus is on a one-consumer economy, in which the individual consumer's indirect utility function coincides with the representative consumer's indirect utility function, and the change in welfare is derived. In a many-consumer economy, with heterogeneous consumers, these analyses are inapplicable. This is because the aggregated compensating variation may not be consistent with the compensation test and thus may be of limited use. This is known as the Boadway paradox.[6] Our analysis is complementary to existing analyses in that we can aggregate each consumer's welfare change consistently but the form of the indirect utility functions is limited to the Gorman form.[7]

The third line of research is recent empirical applications of discrete choice models, which range from models for durable goods such as housing (Earnhart 2002) and automobiles (Berry et al. 1995, Goldberg 1995, and Petrin 2002), to daily consumables such as ready-to-eat cereal (Nevo 2002) and tuna (Nevo and Hatzitaskos 2006). The point is whether it is appropriate to assume that consumers choose no more than one unit of a good. The validity of this assumption depends on the characteristics of the good. Arguably, the assumption is reasonable for housing and automobiles but not for ready-to-eat cereal and tuna because different consumers demand different amounts. We establish a theoretical basis for applying discrete choice models to cases in which consumers choose multiple brands and in which different consumers demand different amounts.

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[5] See Pakes et al. (1993), Berry et al. (1999), and Petrin (2002) for empirical research in which it is assumed that the utility obtained from a good is nonlinear with respect to income.

[6] See Boadway (1974).

[7] Blackorby and Donaldson (1990) show that the Boadway paradox is resolved by assuming that the representative consumer has an indirect utility function of the Gorman form.

The rest of the paper is structured as follows. In Section 2, we focus on the logit model. In Section 3, the analysis is extended to the GEV model. In Section 4, we examine the mixed logit model. In Section 5, we examine the mixed GEV model. Section 6 concludes the paper.

2. The Logit Model

We begin with the logit model. The GEV model, which includes the logit model as a special case, is discussed in the next section.

Consider an N-consumer economy with $M+1$ goods. The goods are numbered consecutively from 0 to M. The price of good 0, whose market demand is X_0, is normalized at unity. The market demand of good j ($j=1,...,M$), X_j, is assumed to be consistent with the result of the discrete choice, i.e., it has the form of the total demand times the choice probability. The price of good j is p_j. The income of consumer i is y^i ($i=1,...,N$).

Utility maximization by consumer i yields the indirect utility function, $v^i(p_1,..., p_M, y^i)$. In this paper, we assume that each consumer's preference can be aggregated to a representative consumer's preference. Without this assumption, there is no clear relationship between the sum of consumers' compensating variations and the compensation principle, as Blackorby and Donaldson (1990) show.

Gorman (1961) shows that in order to define the preferences of a representative consumer by aggregating individual consumer preferences, consumer i's indirect utility function must have the so-called Gorman form:

$$(1) \quad v^i(p_1,..., p_M, y^i) = A^i(p_1,..., p_M) + B(p_1,..., p_M)y^i .$$

Henceforth, we refer to this requirement as the Gorman restriction. Summing indirect utility functions across consumers yields the representative consumer's indirect utility function, as follows:

(2) $$V = \sum_{i=1}^{N} v^i(p_1,..., p_M, y^i) = \sum_{i=1}^{N} A^i(p_1,..., p_M) + B(p_1,..., p_M)Y,$$

where $Y \equiv \sum_{i=1}^{N} y^i$ is aggregate income.

2.1 A utility maximization problem that yields logit-type demand functions

For now, suppose that all goods belong to the same group. The case of different groups is addressed in Section 2.4. Suppose here that the market demand function for good j is consistent with the logit model:

(3) $$X_j(p_1,..., p_M, Y) = C(p_1,..., p_M, Y)s_j(p_1,..., p_M, Y),$$

where $C(p_1,..., p_M, Y)$ is the market demand for a 'group', which is the sum of demands for goods 1 to M, and $s_j(p_1,..., p_M, Y)$ is the choice probability from the logit model:

(4) $$s_j(p_1,..., p_M, Y) = \frac{\exp(u_j(p_j, Y))}{\sum_{k=1}^{M} \exp(u_k(p_k, Y))}.$$

$u_j(p_j, Y)$ corresponds to the utility of alternative j in discrete choice literature, but all we need here is that u_j is a function of p_j and Y whatever interpretation you may make.

We obtain the following proposition about the form of the indirect utility function of the representative consumer.

Proposition 1

The necessary and sufficient condition for the market demand function for good j to have the form of (3) is that the indirect utility function of the representative consumer be:

(5) $V = A(LS) + B(LS)Y$,

where $A(LS) \equiv \sum_{i=1}^{N} A^{i}(p_1,..., p_M)$ and:

(6) $LS \equiv -\frac{1}{\beta}\ln\sum_{k=1}^{M}\exp(\alpha_k - \beta p_k)$.

The market demand function for good j , (3), satisfies:

(7) $C(p_1,..., p_M, Y) = C(LS, Y) = \frac{-\left(\frac{\partial A}{\partial LS} + \frac{\partial B}{\partial LS}Y\right)}{B} > 0$,

(8) $s_j(p_1,..., p_M) = \frac{\exp(\alpha_j - \beta p_j)}{\sum_{k=1}^{M}\exp(\alpha_k - \beta p_k)} = \frac{\partial LS}{\partial p_j}$.

Proof

See Appendix 1.

A model in which a total demand is exogenously fixed is ‘partial’, because the determination of the total demand is left unexplained. Because the total demand is endogenously determined, the representative consumer’s indirect utility function, (5), corresponds to complete utility maximization; the market demand for a group, (7), is endogenously determined by the log-sum term, (6) and by aggregate income. Thus, although the utility maximization problem described in Proposition 1 yields a result that is consistent with the logit model, it does not suffer from the problem associated with discrete choice models identified by McFadden (1999, p. 273), Nevo (2000, footnote 14), and Nevo (2001, footnote 13); that is, the problem of having to assume an a priori total demand. The representative consumer's indirect utility function, (5), includes the case of the fixed total demand as a special case; for instance, $V = -NLS + Y$ represents a representative consumer's indirect utility function that is consistent with the logit model in which the total demand is N.

The market demand for a group, (7), is a function of the log-sum term and aggregate income only; it depends on the prices of each good only through the log-sum term. The log-sum term plays the role of the aggregate price or the price index for the market demand for the group of goods. This property is of practical use for the estimation of demand structures. Suppose that one estimates a logit model and that, as a next step, one estimates the market demand for a group by using the estimated log-sum term. In this case, the derived market demand for a group of goods and the demand for each individual good are consistent with a complete utility maximization model.

The term $u_j(p_j, Y)$, which can be interpreted as the utility obtained from consuming a unit of good j, must be linear in price and independent of income; that is, $u_j = \alpha_j - \beta p_j$. If it is nonlinear in price, the market demand for a group differs among goods, which contradicts the fact that the logit-type market demand function for a good has the form of the common market demand for a group multiplied by the choice probability for a good. If the utility from consuming a unit of good j depends on income, the log-sum term also depends on income and, consequently, the Gorman restriction is not satisfied; hence, aggregating consumers' preferences to those of the representative consumer is impossible.

Note that, in the logit model, the utility from consuming a unit of good j and, thus, the choice probability for good j, are the same for all consumers. (See Appendix 1 for details.) When the utility from consuming a unit of good j and, thus, the choice probability for good

j differs among consumers, the corresponding model is the mixed logit model, which is analyzed in Section 4.

Anderson et al. (1988, 1992 Ch. 3) derive the direct utility function of a representative consumer when the market demand for a group of goods is endogenously determined. In our framework, the corresponding direct utility function is:

$$(9)\quad U = X_0 + h\left(\frac{b}{\beta}\sum_{k=1}^{M} X_k\right) + \frac{1}{\beta}\sum_{k=1}^{M}\left[\alpha_k - \ln\left(\frac{X_k}{\sum_{k=1}^{M} X_k}\right)\right]X_k ,$$

where b is a constant, $h' > 0$, and $h'' < 0$. This direct utility function corresponds to a special case of (5), as we show below. Maximizing (9) with respect to the budget constraint:

$$(10)\quad Y = X_0 + \sum_{k=1}^{M} p_k X_k ,$$

yields the following market demand function for good j:

$$(11)\quad X_j = \frac{\beta}{b} h'^{-1}\left(\frac{\beta}{b} LS\right) \frac{\exp(\alpha_j - \beta p_j)}{\sum_{k=1}^{M} \exp(\alpha_k - \beta p_k)} .$$

Substituting (11) into (9) yields the representative consumer's indirect utility function:

$$(12)\quad V = h\left(h'^{-1}\left(\frac{\beta}{b} LS\right)\right) - \frac{\beta}{b} LS h'^{-1}\left(\frac{\beta}{b} LS\right) + Y .$$

Eq. (12) is a special case of (5), in which $A = h\left(h'^{-1}\left(\frac{\beta}{b} LS\right)\right) - \frac{\beta}{b} LS h'^{-1}\left(\frac{\beta}{b} LS\right)$ and $B = 1$.

The conditions that $h' > 0$ and $h'' < 0$ are sufficient for $\frac{\partial X_j}{\partial p_j} < 0$, because:

$$(15)\quad \frac{\partial X_j}{\partial p_j} = \frac{-\beta^2 \exp(\alpha_j - \beta p_j)\left\{ bh'^{-1}(Q)\sum_{k\neq j}\exp(\alpha_k - \beta p_k) - h''^{-1}(Q)\exp(\alpha_j - \beta p_j)\right\}}{\left(b\sum_{k=1}^{M}\exp(\alpha_k - \beta p_k)\right)^2}.$$

2.2 Elasticities

From the market demand function (3), we obtain the following proposition regarding elasticities.

Proposition 2

The own-price elasticity from the logit-type market demand function for good j is:

$$(14)\quad \frac{\partial X_j}{\partial p_j}\frac{p_j}{X_j} = \theta_j - \beta(1 - s_j)p_j,$$

where $\theta_j \equiv \frac{\partial C}{\partial p_j}\frac{p_j}{C}$ is the elasticity of the market demand for a group of goods, C, with respect to the price of good j, p_j. The cross-price elasticity of demand for good j is:

$$(15)\quad \frac{\partial X_j}{\partial p_{j'}}\frac{p_{j'}}{X_j} = \theta_{j'} + \beta s_{j'} p_{j'},$$

where $j' = 1,...,M$ and $j' \neq j$.

Proof

These results follow straightforwardly from the market demand function, (3).

Both elasticities differ from those in classical discrete choice models by adding the price elasticity of market demand for the group. The cross-price elasticities are the same among all goods, and the property of independence from irrelevant alternatives (IIA) holds, even if the market demand for a group is endogenous.

With regard to the own-price elasticity, we cannot determine whether demand is more elastic if the market demand for a group is exogenous or endogenous. For example, suppose that the market demand for a group is estimated by assuming that it is exogenously fixed and includes the three choices 'select A', 'select B', or 'select neither'. The choice probability is one-third for each alternative. For the sake of simplicity, assume that $\beta p_A = 1$. The own-price elasticity of X_A is $\frac{\partial X_A}{\partial p_A}\frac{p_A}{X_A} = -\frac{2}{3}$ from (14). When the market demand for a group is endogenous, the true own-price elasticity of X_A is $\frac{\partial X_A}{\partial p_A}\frac{p_A}{X_A} = \theta_A - \frac{1}{2}$ from $s_A = s_B = \frac{\frac{1}{3}}{\frac{1}{3}+\frac{1}{3}} = \frac{1}{2}$. $\theta_A - \frac{1}{2}$ may be larger or smaller than $-\frac{2}{3}$, depending on the value of θ_A.

With regard to the cross-price elasticity, we derive a clearer result; ignoring the price elasticity of market demand for a group may change the sign of the cross-price elasticity of each good. If the market demand for a group is exogenously fixed, the cross-price elasticity of good j is positive because $\beta s_{j'} p_{j'} > 0$. By contrast, if market demand is endogenous, the cross-price elasticity of good j may be negative because $\theta_{j'} < 0$. In reality, when the market demand for a group is endogenous, an increase in the price of a particular good has the twin effects of increasing demand for substitutes and decreasing the market demand for the group. Fixing the market demand for a group eliminates the latter effect, and, therefore, an increase in the price of a particular good cannot lower demand for other goods.

2.3 Welfare analysis

In this section, we focus on calculating equivalent variation. The same procedure applies for calculating compensating variation if V^{WO} and v^{WO} are substituted for V^{W} and v^{W}, respectively, in the following analysis. Henceforth, the superscripts WO and W denote without and with a policy, respectively. The results are summarized in Proposition 3.

Proposition 3

Equivalent variation can be calculated from the consumer's demand for a good, the market demand for a good, each consumer's demand for a group of goods, or the market demand for a group of goods, as follows:

$$
\begin{aligned}
EV &= \sum_{i=1}^{N} \int_{p_j^W}^{p_j^{WO}} h_j^i(LS, p_1, \ldots, p_M, v^{iW}) dp_j \\
&= \int_{p_j^W}^{p_j^{WO}} H_j(LS, V^W) dp_j \\
&= \int_{LS^W}^{LS^{WO}} \sum_{i=1}^{N} c^i(LS, p_1, \ldots, p_M, v^{iW}) dLS \\
&= \int_{LS^W}^{LS^{WO}} C(LS, V^W) dLS,
\end{aligned}
\tag{16}
$$

where $h_j^i(LS, p_1, \ldots, p_M, v^i)$ is consumer i's Hicksian demand function for good j, $H_j(LS, V)$ is the Hicksian market demand function for good j, $c^i(LS, p_1, \ldots, p_M, v^i)$ is consumer i's Hicksian demand function for a group of goods, and $C(LS, V)$ is the Hicksian market demand function for a group of goods.

Proof

See Appendix 2.

The transformation from the first to the second line and from the third to the fourth line in (16) follows from the fact that, under the Gorman restriction, each consumer's indirect utility function can be aggregated to the indirect utility function of the representative consumer. The distinctive feature of the logit model, which is also true of GEV models (as shown later), is that the equivalent variation can be calculated not only from the demand and price of good j but also from the demand for a group of goods, by using the log-sum term. In particular, if the market demand of a group is equal to the number of consumers, N, the result in (16) reduces to the well-known method developed by Small and Rosen (1981), under which equivalent variation is calculated from the change in the log-sum term multiplied by the number of consumers.

2.4 Multiple groups

We have so far assumed that goods $1,...,M$ belong to the same group. In reality, goods can be classified into multiple groups, such as food and clothes. Our analysis can be readily extended to the case in which goods are classified into multiple groups and a consumer can choose multiple goods from multiple groups.

Suppose that the goods are classified into G groups and that good j belongs to group g ($g=1,...,G$), without loss of generality. Suppose that the market demand function for good j, which belongs to group g, is consistent with the logit model, as follows:

$$(17)\quad X_j(p_1,...,p_M,Y)=C_g(p_1,...,p_M,Y)s_{gj}(p_1,...,p_M,Y),$$

where $s_{gj}(p_1,...,p_M,Y)=\dfrac{\exp(u_j(p_j,Y))}{\sum_{k\in g}\exp(u_k(p_k,Y))}$ is the choice probability for good j within group g and $C_g(p_1,...,p_M,Y)$ is the market demand for group g.

Propositions 1 to 3 require only minor modification when there are multiple groups. To avoid repetition, our analysis of multiple groups is relegated to Appendix 3, in which we derive Propositions 1' to 3', which are modified versions of Propositions 1 to 3.

Because the relationship between groups is unrestricted, we can represent various relationships between groups in our model. This is a clear advantage over a typical discrete choice model that incorporates the grouping of goods, such as the nested logit model, which limits the relationship between groups to the logit.

As an example, suppose that there are two groups ($l=1,2$) and that preferences between them are represented by the CES utility function. Suppose further that the preference within each group is of the logit type. The indirect utility function of the representative consumer is:

(18) $V=Y\left(\sum_{l=1}^{2} LS_l^{1-\sigma}\right)^{\frac{1}{\sigma-1}}$,

where $LS_g=-\frac{1}{\beta}\ln\sum_{k\in g}\exp(\alpha_k-\beta_g p_k)$ and σ is the elasticity of substitution between the two groups. The demand for group g, given by C_g, has the CES form as follows:

(19) $C_g=\frac{YLS_g^{-\sigma}}{\sum_{l=1}^{2} LS_l^{1-\sigma}}$.

The market demand function for good j in group g is:

(20) $X_j=\frac{YLS_g^{-\sigma}}{\sum_{l=1}^{2} LS_l^{1-\sigma}}\frac{\exp(\alpha_j-\beta_g p_j)}{\sum_{k\in g}\exp(\alpha_k-\beta_g p_k)}$.

This has the form of the CES-type market demand function for a group multiplied by the logit-type choice probability.

3. The GEV Model

The analysis of Section 2 can be extended to the GEV model, which is a general form of logit model. From McFadden (1978, Theorem 1), the GEV model can be described by using the function $F(z_1,\cdots,z_M)$, where $z_j\equiv\exp(u_j(p_j,Y))$.

(GEV–1) $F(z_1,\cdots,z_M)$ is nonnegative.

(GEV–2) $F(z_1,\cdots,z_M)$ is homogenous of degree n.[9]

(GEV–3) $\lim_{z_j\to\infty} F(z_1,\cdots,z_M)=\infty$.

(GEV–4) The ς th partial derivative of $F(z_1,\cdots,z_M)$ with respect to any combination of distinct z_j s is nonnegative if ς is odd and nonpositive if ς is even. That is, $\frac{\partial F}{\partial z_j}\geq 0$ for all j, $\frac{\partial^2 F}{\partial z_j \partial z_{j'}}\leq 0$ for all $j'=1,...,M$ and $j'\neq j$, $\frac{\partial^3 H}{\partial z_j \partial z_{j'} \partial z_{j''}}\geq 0$ for any distinct j, j', and j'' ($j''=1,...,M$), and so on for higher-order derivatives.

Under assumptions (GEV–1) to (GEV–4), from McFadden (1978, Theorem 1), the choice probability for good j is:

$$(21)\quad s_{GEVj}(p_1,...,p_M)=\frac{\frac{\partial F}{\partial z_j}z_j}{nF}.$$

Extending the analysis of Section 2 to the GEV model is straightforward. The points to note are as follows.

i) Proposition 1 holds if the log-sum term and the choice probability for good j are modified from (6) and (8) to:

$$(22)\quad LS_{GEV}\equiv -\frac{1}{n\beta}\ln F(\exp(\alpha_1-\beta p_1),\cdots,\exp(\alpha_M-\beta p_M)),$$

[9] McFadden (1978, Theorem 1) assumed homogeneity of degree one. Ben-Akiva and Francois (1983) demonstrate that H can be homogeneous of degree n. See also Ben-Akiva and Lerman (1985, p. 126).

(23) $$s_{GEVj}(p_1,\ldots,p_M)=\frac{\frac{\partial F}{\partial z_j}\exp(\alpha_j-\beta p_j)}{nF}=\frac{\partial LS_{GEV}}{\partial p_j}.$$

ii) The essence of Proposition 2 holds: for the GEV model, the elasticities of market demand are added to the standard own-price and cross-price elasticities when the market demand for a group is endogenous. The own-price elasticity of market demand for good j, X_{GEVj}, is:

(24) $$\frac{\partial X_{GEVj}}{\partial p_j}\frac{p_j}{X_{GEVj}}=\theta_j-\beta(1-ns_{GEVj})p_j+\eta_{jj},$$

where $\eta_{jj}\equiv\frac{\partial\left(\frac{\partial F}{\partial z_j}\right)}{\partial p_j}\frac{p_j}{\left(\frac{\partial F}{\partial z_j}\right)}$ is the elasticity of $\frac{\partial F}{\partial z_j}$ with respect to the price of good j, p_j.

The cross-price elasticity is:

(25) $$\frac{\partial X_{GEVj}}{\partial p_{j'}}\frac{p_{j'}}{X_{GEVj}}=\theta_{j'}+\beta ns_{j'}p_{j'}+\eta_{jj'}.$$

iii) With regard to welfare analysis, Proposition 3 holds, although the log-sum term is modified from (16) to (22).

iv) The extension to the case of multiple groups is analogous to that for the logit model.

4. The Mixed Logit Model

From now on, we focus on ‘mixed’ versions of the logit and GEV models. We consider the mixed logit model in this section and consider the mixed GEV model in the next section.

Suppose that each consumer derives a different level of utility from consuming a unit of good j. The differences in utility among consumers are unobservable and are treated probabilistically; we assume that each consumer has his or her own parameter, γ^i, whose probability density function is $f(\gamma^i)$.

From Train (2003), the mixed logit model is defined as the model that has the following choice probability:

$$(26)\ \ s_{ML\,j} = \int_{\gamma^i} s^i_{MLj}(p_1,...,p_M, y^i, \gamma^i) f(\gamma^i) d\gamma^i,$$

where:

$$(27)\ \ s^i_{MLj}(p_1,...,p_M, y^i, \gamma^i) = \frac{\exp(u^i_j(p_j, y^i, \gamma^i))}{\sum_{k=1}^{M} \exp(u^i_k(p_k, y^i, \gamma^i))}.$$

For now, suppose that all goods belong to the same group. Our analysis is easily extended to the case in which goods are classified into multiple groups: see Appendix 6.

Because indirect utility functions must satisfy the Gorman restriction, given that consumer i's parameter is γ^i, the conditional indirect utility function of consumer i is:

$$(28)\ \ v^{ci}(p_1,...,p_M, y^i, \gamma^i) = A^i(p_1,...,p_M, \gamma^i) + B(p_1,...,p_M) y^i.$$

Because γ^i follows the probability density function $f(\gamma^i)$, the unconditional indirect utility function of consumer i is:

$$(29)\ \ v^i = \int_{\gamma^i} v^{ci} f(\gamma^i) d\gamma^i = \int_{\gamma^i} \left(A^i(p_1,...,p_M, \gamma^i) + B(p_1,...,p_M) y^i \right) f(\gamma^i) d\gamma^i.$$

The unconditional indirect utility function of the representative consumer is:

(30)
$$V=\sum_{i=1}^{N}v^i=\sum_{i=1}^{N}\int_{\gamma^i}\left(A^i(p_1,...,p_M,\gamma^i)+B(p_1,...,p_M)y\right)f(\gamma^i)d\gamma^i$$
$$=\sum_{i=1}^{N}\int_{\gamma^i}\left(A^i(p_1,...,p_M,\gamma^i)\right)f(\gamma^i)d\gamma^i+B(p_1,...,p_M)Y.$$

4.1 A utility maximization problem that yields mixed-logit-type demand functions

When a change in consumer i's demand is taken into account, the market demand function for good j that is consistent with the mixed logit model is:

(31)
$$X_{MLj}=\sum_{i=1}^{N}\int_{\gamma^i}c_{ML}^i(p_1,...,p_M,y^i,\gamma^i)s_{MLj}^i(p_1,...,p_M,y^i,\gamma^i)f(\gamma^i)d\gamma^i$$
$$=\sum_{i=1}^{N}\int_{\gamma^i}x_{MLj}^i(p_1,...,p_M,y^i,\gamma^i)f(\gamma^i)d\gamma^i ,$$

where X_{MLj} is the market demand for good j, $c_{ML}^i(p_1,...,p_M,y^i,\gamma^i)$ is consumer i's demand for a group of goods, and $x_{MLj}^i(p_1,...,p_M,y^i,\gamma^i)$ is consumer i's demand for good j.

We can now state the following proposition about the form of the indirect utility function of the representative consumer.

Proposition 4

The necessary and sufficient condition for the market demand function for good j to have the form of (31) is that the indirect utility function of the representative consumer be:

(32) $$V=\sum_{i=1}^{N}\int_{\gamma^i}\left(A^i(LS_{ML}^i(\gamma^i),\gamma^i)\right)f(\gamma^i)d\gamma^i+\overline{B}Y ,$$

where $\overline{B}$ is a fixed constant and:

(33) $$LS_{ML}^i(\gamma^i)\equiv-\frac{1}{\beta^i(\gamma^i)}\ln\sum_{k=1}^{M}\exp(\alpha_k^i(\gamma^i)-\beta^i(\gamma^i)p_k) .$$

The market demand function for good j, (35), satisfies:

$$(34)\ \ c^i_{ML}(LS^i_{ML}(\gamma^i),\gamma^i)=\frac{-\dfrac{\partial A^i}{\partial LS^i_{ML}(\gamma^i)}}{\overline{B}}=\frac{-\dfrac{\partial v^{ci}}{\partial LS^i_{ML}(\gamma^i)}}{\overline{B}}>0,$$

$$(35)\ \ s^i_{MLj}(p_1,...,p_M,\gamma^i)=\frac{\exp(\alpha^i_j(\gamma^i)-\beta^i(\gamma^i)p_j)}{\sum_{k=1}^{M}\exp(\alpha^i_k(\gamma^i)-\beta^i(\gamma^i)p_k)}=\frac{\partial LS^i_{ML}(\gamma^i)}{\partial p_j}.$$

Proof

See Appendix 4.

The major difference from the logit model is that the indirect utility function of the representative consumer is quasi-linear. The reason is as follows. In the mixed logit model, the log-sum term, $LS^i_{ML}(\gamma^i)$, differs among consumers. From Gorman's (1961) restriction, the coefficient of income, $B(p_1,...,p_M)$, must be the same for all consumers. This implies that the coefficient of income, $B(p_1,...,p_M)$, does not include the log-sum term, $LS^i_{ML}(\gamma^i)$. Moreover, one can obtain the market demand function for a group of goods and the choice probability for the mixed logit model only when the coefficient of income, $B(p_1,...,p_M)$, does not depend on the prices, p_j. Thus, $B(p_1,...,p_M)$ is the fixed constant $\overline{B}$, and the indirect utility function of the representative consumer is quasi-linear. Hence, the market demand function for good j is independent of income. The mixed logit model, if it is formulated to be consistent with standard microeconomic utility maximization, can deal with differences in parameters among consumers and can thereby incorporate consumer differences in utility obtained from the consumption of goods. However, the mixed logit model cannot deal with the income effect.

4.2 Elasticities

From the mixed logit-type market demand function, (31), we obtain the following proposition about elasticities.

Proposition 5

The own-price elasticity of the market demand for good j is:

$$(36)\quad \frac{\partial X_{MLj}}{\partial p_j}\frac{p_j}{X_{MLj}} = \sum_{i=1}^{N}\int_{\gamma^i}\left\{\frac{c_{ML}^i s_{MLj}^i}{X_{MLj}}\left(\varphi_j^i - \beta(1-s_{MLj}^i)p_j\right)\right\} f(\gamma^i)d\gamma^i ,$$

where $\varphi_j^i = \frac{\partial c_{ML}^i}{\partial p_j}\frac{p_j}{c_{ML}^i}$ is the price elasticity of consumer i's group demand. The cross-price elasticity is:

$$(37)\quad \frac{\partial X_{MLj}}{\partial p_{j'}}\frac{p_{j'}}{X_{MLj}} = \sum_{i=1}^{N}\int_{\gamma^i}\left\{\frac{c_{ML}^i s_{MLj}^i}{X_{MLj}}\left(\varphi_{j'}^i + \beta s_{MLj'}^i p_{j'}\right)\right\} f(\gamma^i)d\gamma^i .$$

Proof

The results follow straightforwardly from the market demand function, (31).

If the parameter γ^i is a fixed constant, the results in (36) and (37) are consistent with the corresponding results for the logit model, given by (14) and (15), respectively. Otherwise, the own-price and cross-price elasticities are more flexible. Thus, the mixed logit model can deal with more complex substitution and complementarity patterns. In particular, the IIA property does not hold because the cross-price elasticities in (37) depend on $\frac{c_{ML}^i(LS_{ML}^i(\gamma^i),\gamma^i)s_{MLj}^i(p_1,\ldots,p_M,\gamma^i)}{X_{MLj}}$, which differs among goods.

4.3 Welfare analysis

The indirect utility function of the representative consumer that yields the mixed logit model, (32), is quasi-linear. Thus, the Hicksian and the Marshallian demand curves coincide, and equivalent variation, compensating variation, and the change in the consumer surplus also coincide. We can state the following proposition about welfare measurement.

Proposition 6

Equivalent variation can be calculated from consumer i's demand for a good or from consumer i's demand for a group of goods; that is:

$$
\begin{aligned}
EV &= \sum_{i=1}^{N} \int_{\gamma^i} \left(\int_{p_j^{iW}}^{p_j^{iWO}} x_{MLj}^i (LS_{ML}^i(\gamma^i), \gamma^i) dp_j \right) f(\gamma^i) d\gamma^i \\
&= \sum_{i=1}^{N} \int_{\gamma^i} \left(\int_{LS_{ML}^{iW}(\gamma^i)}^{LS_{ML}^{iWO}(\gamma^i)} c_{ML}^i (LS_{ML}^i(\gamma^i), \gamma^i) dLS_{ML}^i(\gamma^i) \right) f(\gamma^i) d\gamma^i .
\end{aligned} \tag{38}
$$

Proof

See Appendix 5.

In the mixed logit model, there are two ways of calculating equivalent variation. The first method is to calculate equivalent variation for consumer i by using his or her demand function for good j, and then sum over all consumers. The second method is to calculate equivalent variation by using his or her demand function for a group of goods and the log-sum term, $LS_{ML}^i(\gamma^i)$. Note that equivalent variation cannot be calculated by using the market demand for good j or the market demand for a group of goods. This represents a difference from Proposition 3 for the logit model. The reason for this is that because consumer i's demand depends on his or her own parameter, it cannot be summed without integrating over this parameter. This result is not surprising given that the mixed logit model explicitly considers differences in parameters among consumers.

As explained in Section 4.1, the mixed logit model cannot deal with the income effect. By contrast, the logit model can deal with the income effect under the Gorman restriction. Thus, welfare measurement based on the logit model, (16), is not generally a special case of that based on the mixed logit model, (38). This is only the case for a quasi-linear indirect utility function. In this case, if all consumers have the same parameter ($\gamma^i = \gamma$), the first and second lines in (38) coincide with the first and third lines in (16).

5. The Mixed GEV Model

McFadden and Train (2000) demonstrate that the mixed logit model can approximate any random utility model. However, the mixed GEV model can be more suitable because of its analytical properties (Bhat et al. 2007). Therefore, in this section, we derive the properties of the mixed GEV model.

The extension from the mixed logit model to the mixed GEV model is analogous to that from the logit model to the GEV model. The following points should be noted.

i) Proposition 4 holds if the log-sum term and the choice probability for good j are modified from (33) and (35) to:

(39) $$LS^i_{MGEV}(\gamma^i) \equiv -\frac{1}{n\beta^i(\gamma^i)} \ln F(\exp(\alpha^i_1(\gamma^i) - \beta^i(\gamma^i)p_1), \cdots, \exp(\alpha^i_M(\gamma^i) - \beta^i(\gamma^i)p_M)),$$

(40) $$s^i_{MGEVj}(p_1, ..., p_M, \gamma^i) = \frac{\frac{\partial F}{\partial z_j}\exp(\alpha^i_j(\gamma^i) - \beta^i(\gamma^i)p_j)}{nG} = \frac{\partial LS^i_{MGEV}(\gamma^i)}{\partial p^i_j}.$$

ii) The own-price and cross-price elasticities from the GEV model are mixed in the same way as are those from the mixed logit model. The own-price elasticity of the market demand for good j, X_{MGEVj}, is:

(41) $$\frac{\partial X_{MGEVj}}{\partial p_j}\frac{p_j}{X_{MGEVj}} = \sum_{i=1}^{N}\int_{\gamma^i}\left\{\frac{c^i_{MGEV}s^i_{MGEVj}}{X_{MGEVj}}\left(\varphi^i_j - \beta(1-ns^i_{MGEVj})p_j + \psi^i_{jj}\right)\right\} f(\gamma^i)d\gamma^i ,$$

where $\psi^i_{jj} \equiv \frac{\partial\left(\frac{\partial F(\gamma^i)}{\partial z^i_j(\gamma^i)}\right)}{\partial p_j}\frac{p_j}{\left(\frac{\partial F(\gamma^i)}{\partial z^i_j(\gamma^i)}\right)}$ is the elasticity of $\frac{\partial F(\gamma^i)}{\partial z^i_j(\gamma^i)}$, where $z^i_j(\gamma^i) \equiv \alpha^i_j(\gamma^i) - \beta^i(\gamma^i)p_j$, with respect to the price of good j, p_j. The cross-price elasticity is:

(42) $$\frac{\partial X_{MGEVj}}{\partial p_{j'}}\frac{p_{j'}}{X_{MGEVj}} = \sum_{i=1}^{N}\int_{\gamma^i}\left\{\frac{c^i_{MGEV}s^i_{MGEVj}}{X_{MGEVj}}\left(\varphi^i_{j'} + \beta ns^i_{MGEVj'}p_{j'} + \psi^i_{jj'}\right)\right\} f(\gamma^i)d\gamma^i .$$

iii) Welfare analysis is the same as in the mixed logit model if $LS^i_{ML}(\gamma^i)$ is replaced by $LS^i_{MGEV}(\gamma^i)$ in (39).

iv) The extension to the case of multiple groups is analogous to that for the mixed logit model.

6. Concluding Remarks

In this paper, we formulated a structure for utility maximization problems that are consistent with demand functions derived from the generalized extreme value (GEV) model and the associated mixed models. We also clarified the characteristics of the form of the utility function, the elasticities, and the measurement of welfare. The results of the paper demonstrate that GEV and mixed GEV models that incorporate endogenous demands for groups of goods; that is, those models without a predetermined total demand, are consistently formulated as standard microeconomic utility maximization problems of a representative consumer. Thus, our analysis shows that the assumption that a consumer can choose only one alternative in discrete choice models can be relaxed, and in that case we do not need defenses by McFadden (1999, p.273) and Nevo (2000, footnote 14 and 2001, footnote 13) anymore. Before concluding our analysis, we comment on three issues.

First, the results of this paper depend on the Gorman restriction. Unfortunately, we cannot conduct practical benefit estimation without the Gorman restriction in a many-consumer economy. Otherwise, we have to derive a social welfare function; but this would be difficult in actual benefit estimation. The Gorman restriction is a cost for practical benefit estimation although it is a severe restriction on the preference. In usual benefit estimation, where the total surplus is the sum of each consumer's surplus, the Gorman restriction is assumed to hold at least implicitly. Under such practical situations, our analysis is fully applicable.

Second, our analysis shows that the utility of each alternative in usual discrete choice models must be independent of income and be linear regarding price. Without this restriction, one cannot relate GEV and mixed GEV models that incorporate endogenous demand for groups of goods to a complete utility maximization problem for a representative consumer. For these cases, we would need another theoretical foundation of discrete choice models based on microeconomics, which is delegated to future research.

Third, when applied to empirical research, our results could lead to richer analysis, because our analysis can incorporate the difference in demand quantity among consumers, maintaining the same form of the demand function as the discrete choice models. For instance, the elasticity of the demand for a group of goods can be separately analyzed from the usual 'choice' elasticity in discrete choice models. The downside of our approach is that we need data on individual consumers. Berry (1994), Berry et al. (1995, 1999), and Nevo (2000, 2001), among others, incorporate a change in the total demand by including outside option. There approach has a great advantage in empirical research, because it can be conducted only with market data. When an individual buys two units of a good, however, their approach assumes that two individuals buy one for each. We need to check the effect of the difference between our approach and outside option approach when applied to actual empirical analyses, which is delegated to future researches.

Appendix 1 Proof of Proposition 1

We prove the necessary condition first. To prove necessity, we need to demonstrate that if the market demand function has the form of (3), the corresponding Gorman-type indirect utility function of the representative consumer is (5), and (6)-(8) are satisfied.

From (2), both $A^i(p_1,...,p_M)$ and $B(p_1,...,p_M)$ depends only on p_j and do not depend on y^i. Define a new variable, $\widetilde{LS}(p_1,...,p_M)$, as

(A1) $$\widetilde{LS}(p_1,...,p_M) \equiv -\ln\sum_{k=1}^{M}\exp(u_k(p_k)).$$

From (A1), any p_j can be represented as a function of $p_{j'}$ and $\widetilde{LS}$. Without loss of generality, we assume $j=1$, and derive p_1 as:

(A2) $$p_1 = p_1(\widetilde{LS}, p_2,..., p_M).$$

Using (A2), we can rewrite the indirect utility function of the representative consumer, (2), as:

(A3) $$\begin{aligned} V &= \sum_{i=1}^{N} v^i(p_1,...,p_M,y^i) = \sum_{i=1}^{N} A^i(p_1(\widetilde{LS}, p_2,...,p_M),...,p_M) + B(p_1(\widetilde{LS}, p_2,...,p_M),...,p_M)Y \\ &= A(\widetilde{LS}, p_2,...,p_M) + B(\widetilde{LS}, p_2,...,p_M)Y, \end{aligned}$$

where $A(\widetilde{LS}, p_2,...,p_M) \equiv \sum_{i=1}^{N} A^i(p_1(\widetilde{LS}, p_2,...,p_M),...,p_M)$.

By applying Roy's Identity to (A3), the market demand function for good 1 is derived as:

(A4) $$X_1 = \sum_{i=1}^{N} x_1^i = \frac{\dfrac{\partial(A+BY)}{\partial\widetilde{LS}} u_1'(p_1) \dfrac{\exp(u_1(p_1))}{\sum_{k=1}^{M}\exp(u_k(p_k))}}{B}.$$

Comparing (3) and (A5) when $j = 1$ reveals:

$$\text{(A5) } C(p_1,..., p_M, Y) = \frac{\frac{\partial(A+BY)}{\partial \widetilde{LS}} u_1'(p_1)}{B} .$$

In the same way, the market demand function for good m ($m = 2,..., M$) is derived as:

$$\text{(A6) } X_m = \frac{\frac{\partial(A+BY)}{\partial \widetilde{LS}} u_m'(p_m) \frac{\exp(u_m(p_m))}{\sum_{k=1}^{M} \exp(u_k(p_k))} - \frac{\partial(A+BY)}{\partial p_m}}{B} .$$

Comparing (3) and (A6) when $j = m$ reveals:

$$\text{(A7) } C(p_1,..., p_M, Y) = \frac{\frac{\partial(A+BY)}{\partial \widetilde{LS}} u_m'(p_m)}{B} ,$$

$$\text{(A8) } \frac{\partial A}{\partial p_m} + \frac{\partial B}{\partial p_m} Y = 0 .$$

Because (A8) holds for any Y, we obtain:

$$\text{(A9) } \frac{\partial A}{\partial p_m} = \frac{\partial B}{\partial p_m} = 0 ,$$

which implies that A and B depend only on $\widetilde{LS}$, that is:

$$\text{(A10)} \qquad V = A(\widetilde{LS}) + B(\widetilde{LS})Y .$$

Because $C(p_1,..., p_M, Y)$ is the same for any good j, from (A5) and (A7), we obtain:

$$\text{(A11)} \qquad u_1'(p_1) = ... = u_M'(p_M) .$$

This implies that $u_j(p_j)$ must be linear in p_j. Thus, we can express $u_j(p_j)$ as:

$$\text{(A12)} \qquad u_j(p_j) = \alpha_j - \beta p_j \,.$$

Including this restriction, the choice probability can be derived as (8), and $\widetilde{LS}$ can be written as:

$$\text{(A13)} \qquad \widetilde{LS} = -\ln \sum_{k=1}^{M} \exp(\alpha_k - \beta p_k) \,.$$

Eq. (6) is derived by defining:

$$\text{(A14)} \qquad LS \equiv \frac{1}{\beta} \widetilde{LS} \,.$$

Replacing $\widetilde{LS}$ in (A10) by LS in (A14) yields (5).

Because the indirect utility function of the representative consumer must be decreasing in p_j, we obtain:

$$\text{(A15)} \qquad \frac{\partial V}{\partial p_j} = \frac{\partial V}{\partial LS} \frac{\partial LS}{\partial p_j} = \frac{\partial V}{\partial LS} \frac{\exp(\alpha_j - \beta p_j)}{\sum_{k=1}^{M} \exp(\alpha_k - \beta p_k)} < 0 \,.$$

This implies:

$$\text{(A16)} \qquad \frac{\partial V}{\partial LS} = \frac{\partial (A + BY)}{\partial LS} < 0 \,.$$

From (A7), (A14), (A16), and $u_1'(p_1) = ... = u_M'(p_M) = -\beta < 0$, the market demand for a group of goods, (7), is:

(A17) $$C(LS,Y)=\frac{\frac{\partial(A+BY)}{\partial\widetilde{LS}}u_m{}'(p_m)}{B}=\frac{-\frac{\partial(A+BY)}{\partial LS}}{B}>0\,.$$

By using Roy's Identity, proving sufficiency is straightforward. Applying Roy's Identity to (5) yields:

(A18) $$X_j=\frac{-\frac{\partial V}{\partial p_j}}{\frac{\partial V}{\partial Y}}=\frac{-\frac{\partial(A+BY)}{\partial LS}}{B}\frac{\exp(\alpha_j-\beta p_j)}{\sum_{k=1}^{M}\exp(\alpha_k-\beta p_k)}\,,$$

where:

(A19) $$C(LS,Y)=\frac{-\frac{\partial(A+BY)}{\partial LS}}{B}\,.$$

From (A16), this expression is positive. In addition:

(A20) $$s_j(p_1,...,p_M)=\frac{\exp(\alpha_j-\beta p_j)}{\sum_{k=1}^{M}\exp(\alpha_k-\beta p_k)}=\frac{\partial LS}{\partial p_j}\,.$$

Eq. (3) is derived from (A18) to (A20).

Appendix 2 Proof of Proposition 3

From (5), the expenditure function of the representative consumer is:

(A21) $$E(LS,V)=\frac{V-A(LS)}{B(LS)}\,.$$

From (A21), we obtain the Hicksian market demand function for good j :

(A22) $$H_j(LS,V)=\sum_{i=1}^{N}h_j=\frac{\partial E}{\partial p_j}=\frac{\left(-\frac{\partial A}{\partial LS}B-(V-A)\frac{\partial B}{\partial LS}\right)}{(B)^2}\frac{\partial LS}{\partial p_j},$$

where h_j is consumer i 's Hicksian demand function for good j .

From (A21), equivalent variation, EV , can be calculated as:

(A23)
$$\begin{aligned}EV&=\int_{p_j^W}^{p_j^{WO}}H_j(LS,V^W)dp_j\\&=\int_{p_j^W}^{p_j^{WO}}\frac{-\left(\frac{\partial A}{\partial LS}\frac{\partial LS}{\partial p_j}\right)B-(V-A)\frac{\partial B}{\partial LS}\frac{\partial LS}{\partial p_j}}{(B)^2}dp_j\\&=\int_{LS^W}^{LS^{WO}}\frac{-\frac{\partial A}{\partial LS}B-(V-A)\frac{\partial B}{\partial LS}}{(B)^2}dLS\\&=\int_{LS^W}^{LS^{WO}}\sum_{j=1}^{M}H_j(LS,V)dLS.\end{aligned}$$

Denoting the total demand of consumer i and the representative consumer by C and c^i respectively, we obtain:

(A24) $$C(LS,V^W)=\sum_{j=1}^{M}H_j(LS,V)=\sum_{i=1}^{N}c^i(p_1,...,p_M,v^{iW}).$$

From (A22) and (A23), we obtain (16).

Appendix 3 Analysis for Multiple Groups

Taking into account the classification into multiple groups, Propositions 1 to 3 are modified as follows.

Proposition 1'

The necessary and sufficient condition for the market demand function for good j to have the form of (17) is that the indirect utility function of the representative consumer be:

(A25) $$V = A(LS_1,...,LS_G) + B(LS_1,...,LS_G)Y\ ,$$

where $A(LS_1,...,LS_G) \equiv \sum_{i=1}^{N} A^i(p_1,...,p_M)$ and:

(A26) $$LS_g \equiv -\frac{1}{\beta_g}\ln\sum_{k\in g}\exp(\alpha_k - \beta_g p_k)\ .$$

The market demand function for good j, (17), satisfies:

(A27) $$C_g(LS_1,...,LS_G,Y) = \frac{-\dfrac{\partial(A+BY)}{\partial LS_g}}{B} = \frac{-\dfrac{\partial V}{\partial LS_g}}{B} > 0\ ,$$

(A28) $$s_{gj}(...,p_k,...) = \frac{\exp(\alpha_j - \beta_g p_j)}{\sum_{k\in g}\exp(\alpha_k - \beta_g p_k)} = \frac{\partial LS_g}{\partial p_j}\ .$$

Proposition 2'

The own-price elasticity of the market demand for good j, X_j, is:

(A29) $$\frac{\partial X_j}{\partial p_j}\frac{p_j}{X_j} = \theta_{gj} - \beta_g(1 - s_{gj})p_j\ ,$$

where $\theta_{gj} \equiv \frac{\partial C_g}{\partial p_j}\frac{p_j}{C_g}$ is the elasticity of the market demand for group g, C_g, with respect to the price of good j, p_j. The cross-price elasticity is:

(A30) $$\frac{\partial X_j}{\partial p_{j'}}\frac{p_{j'}}{X_j} = \theta_{gj'} + \beta_g s_{gj'} p_{j'}\ ,\ \text{where}\ \theta_{gj'} \equiv \frac{\partial C_g}{\partial p_{j'}}\frac{p_{j'}}{C_g}\ ,$$

when both goods j and j' belong to the same group. When these goods belong to different groups, the corresponding cross-price elasticity is:

(A31) $$\frac{\partial X_j}{\partial p_{j'}} \frac{p_{j'}}{X_j} = \theta_{gj'} .$$

Proposition 3'

Equivalent variation can be calculated from the consumer's demand for a good, the market demand for a good, the consumer's demand for a group of goods, or the market demand for a group of goods, as follows:

(A32) $$\begin{aligned} EV &= \sum_{i=1}^{N} \int_{p_j^W}^{p_j^{WO}} h_j^i (LS_1,..., LS_G, p_1,..., p_M, v^{iW}) dp_j \\ &= \int_{p_j^W}^{p_j^{WO}} H_j (LS_1,..., LS_G, V^{iW}) dp_j \\ &= \int_{LS_g^W}^{LS_g^{WO}} \sum_{i=1}^{N} c_g^i (LS_1,..., LS_G, p_1,..., p_M, v^{iW}) dLS_g \\ &= \int_{LS_g^W}^{LS_g^{WO}} C_g (LS_1,..., LS_G, V^{iW}) dLS_g . \end{aligned}$$

where $c_g^i (LS_1,..., LS_G, p_1,..., p_M, v^{iW})$ is consumer i 's Hicksian demand for group g , and $C_g (LS_1,..., LS_G, V^{iW})$ is the Hicksian market demand for group g.

The derivation of the above results is a straightforward extension of the analysis of Sections 2.1 to 2.3. A different result is that goods j and j' may belong to different groups, in which case, (A31) holds: the cross-price elasticity depends only on the elasticity of the market demand for group g with respect to the price of good j , $\theta_{gj'}$.

Appendix 4 Proof of Proposition 4

The steps of proof are the same as Proposition 1. We prove the necessary condition first. To prove necessity, we need to demonstrate that if the market demand function has the form of (31), the corresponding Gorman-type indirect utility function of the representative consumer is (32), and (33)-(35) are satisfied. From (30), $A^i(p_1,...,p_M,\gamma^i)$ is a function of p_j and γ^i and $B(p_1,...,p_M)$ is a function of p_j. Define a new variable, $\widetilde{LS^i_{ML}}(p_1,...,p_M,\gamma^i)$, as:

(A33) $$\widetilde{LS^i_{ML}}(p_1,...,p_M,\gamma^i) \equiv -\ln\sum_{k=1}^{M}\exp(u^i_k(p_k,\gamma^i)).$$

From (A33), any p_j can be represented as a function of $p_{j'}$, $\widetilde{LS}$, and γ^i. Without loss of generality, we assume $j=1$, and derive p_1 as:

(A34) $$p_1 = p_1(\widetilde{LS^i_{ML}}, p_2,...,p_M,\gamma^i).$$

Using (A34), we can rewrite the indirect utility function of the representative consumer, (30), as:

(A35)

$$V = \sum_{i=1}^{N}\int_{\gamma^i}\left(A^i(p_1(\widetilde{LS^i_{ML}}, p_2,...,p_M,\gamma^i),...,p_M,\gamma^i)\right) f(\gamma^i)d\gamma^i + B(p_1(\widetilde{LS^i_{ML}}, p_2,...,p_M,\gamma^i),...,p_M)Y .$$

One of the following conditions must be met for (A35) to satisfy the Gorman restriction:

i) $\widetilde{LS^i_{ML}}(p_1,...,p_M,\gamma^i) = \widetilde{LS_{ML}} = -\ln\sum_{k=1}^{M}\exp(u_k(p_k))$,

ii) $B(p_1(\widetilde{LS^i_{ML}}, p_2,...,p_M,\gamma^i),...,p_M) = B(p_2,...,p_M)$.

In case i), the choice probability degenerates to that in the logit model. Thus, we focus on case ii), where (A35) is rewritten as:

(A36) $$V = \sum_{i=1}^{N}\int_{\gamma^i}\left(A^i(\widetilde{LS^i_{ML}}, p_2,...,p_M,\gamma^i)\right) f(\gamma^i)d\gamma^i + B(p_2,...,p_M)Y .$$

Applying Roy's Identity to (A36) yields the market demand function for good 1:

(A37) $$X_{ML1} = \sum_{i=1}^{N} \int_{\gamma^i} \left(\frac{\frac{\partial A^i}{\partial \widetilde{LS_{ML}^i}} \frac{\partial u_1^i(p_1, \gamma^i)}{\partial p_1}}{B(p_2, ..., p_M)} \left(\frac{\exp(u_1^i(p_1, \gamma^i))}{\sum_{k=1}^{M} \exp(u_k^i(p_k, \gamma^i))} \right) \right) f(\gamma^i) d\gamma^i .$$

Comparing (31) and (A37) when $j = 1$ reveals:

(A38) $$c^i(p_1, ..., p_M, \gamma^i) = \frac{\frac{\partial A^i}{\partial \widetilde{LS_{ML}^i}} \frac{\partial u_1^i(p_1, \gamma^i)}{\partial p_1}}{B(p_2, ..., p_M)} .$$

Analogously, the market demand function for good m ($m = 2, ..., M$) is:

(A39) $$X_{MLm} = \sum_{i=1}^{N} \int_{\gamma^i} \left(\frac{\frac{\partial A^i}{\partial \widetilde{LS_{ML}^i}} \frac{\partial u_m^i(p_m, \gamma^i)}{\partial p_m} \left(\frac{\exp(u_m^i(p_m, \gamma^i))}{\sum_{k=1}^{M} \exp(u_k^i(p_k, \gamma^i))} \right) - \left(\frac{\partial A^i}{\partial p_m} + \frac{\partial B}{\partial p_m} y^i \right)}{B(p_2, ..., p_M)} \right) f(\gamma^i) d\gamma^i .$$

Comparing (31) and (A39) when $j = m$ reveals:

(A40) $$c^i(p_1, ..., p_M, \gamma^i) = \frac{\frac{\partial A^i}{\partial \widetilde{LS_{ML}^i}} \frac{\partial u_m^i(p_m, \gamma^i)}{\partial p_m}}{B(p_2, ..., p_M)} ,$$

(A41) $$\sum_{i=1}^{N} \int_{\gamma^i} \left(\frac{\frac{\partial A^i}{\partial p_m} + \frac{\partial B}{\partial p_m} y^i}{B(p_2, ..., p_M)} \right) f(\gamma^i) d\gamma^i = 0 .$$

From (A36), we obtain:

(A42) $$\frac{\partial A^i}{\partial p_1} = \frac{\partial B}{\partial p_1} = 0,$$

which is the result of rewriting p_1 as $p_1 = p_1(\widetilde{LS^i_{ML}}, p_2, ..., p_M, \gamma^i)$ in (A34). Since the result must be independent of the choice of p_j to be rewritten, we have:

(A43) $$\frac{\partial A^i}{\partial p_m} = \frac{\partial B}{\partial p_m} = 0.$$

in (A41). (A42) and (A43) imply that the indirect utility function of the representative consumer is independent of p_j, as follows:

(A44) $$V = \sum_{i=1}^{N} \int_{\gamma^i} \left(A^i(\widetilde{LS^i_{ML}}(p_1, ..., p_M, \gamma^i), \gamma^i) \right) f(\gamma^i) d\gamma^i + \overline{B}Y .$$

where $\overline{B}$ is constant.

Because $c^i(p_1, ..., p_M, \gamma^i)$ is the same for all j, from (A37) and (A39) we obtain:

(A45) $$\frac{\partial u^i_1(p_1, \gamma^i)}{\partial p_1} = ... = \frac{\partial u^i_m(p_m, \gamma^i)}{\partial p_m} .$$

This expression indicates that $\frac{\partial u^i_j(p_j, \gamma^i)}{\partial p_j}$ is linear in p_j. Thus, we can express $u^i_j(p_j, \gamma^i)$ as:

(A46) $$u^i_j(p_j, \gamma^i) = \alpha^i_j(\gamma^i) - \beta^i(\gamma^i) p_j .$$

The choice probability, (35), follows from (27) and (A46). Given (A46), $\widetilde{LS^i_{ML}}(p_1, ..., p_M, \gamma^i)$ can be written as:

(A47) $$\widetilde{LS^i_{ML}}(p_1,...,p_M,\gamma^i) = -\ln\sum_{k=1}^{M}\exp(\alpha^i_k(\gamma^i)-\beta^i(\gamma^i)p_k).$$

Eq. (33) is derived by defining:

(A48) $$LS^i_{ML}(\gamma^i) \equiv \frac{1}{\beta^i(\gamma^i)}\widetilde{LS^i_{ML}}(p_1,...,p_M,\gamma^i).$$

Replacing $\widetilde{LS^i_{ML}}$ in (A44) by $LS^i_{ML}(\gamma^i)$ in (A48) yields (32).

Because consumer i's conditional indirect utility function must be decreasing in p_j, we obtain:

(A49) $$\frac{\partial v^{ci}}{\partial p_j} = \frac{\partial v^{ci}}{\partial LS^i_{ML}(\gamma^i)}\frac{\partial LS^i_{ML}(\gamma^i)}{\partial p_j} = \frac{\partial v^{ci}}{\partial LS^i_{ML}(\gamma^i)}\frac{\exp(\alpha^i_j(\gamma^i)-\beta^i(\gamma^i)p_j)}{\sum_{k=1}^{M}\exp(\alpha^i_k(\gamma^i)-\beta^i(\gamma^i)p_k)} < 0.$$

This implies:

(A50) $$\frac{\partial v^{ci}}{\partial LS^i_{ML}(\gamma^i)} = \frac{\partial A^i}{\partial LS^i_{ML}(\gamma^i)} < 0.$$

Given (A40), (A48), (A50), and $\frac{\partial u^i_1(p_1,\gamma^i)}{\partial p_1} = ... = \frac{\partial u^i_m(p_m,\gamma^i)}{\partial p_m} = -\beta^i(\gamma^i)$, consumer i's demand for a group of goods, (34), is:

(A51) $$c^i_{ML}(LS^i_{ML}(\gamma^i),\gamma^i) = \frac{\frac{\partial A^i}{\partial \widetilde{LS^i_{ML}}}u^{i\,\prime}_m(p^i_m)}{\overline{B}} = \frac{-\frac{\partial A^i}{\partial LS^i_{ML}(\gamma^i)}}{\overline{B}} > 0.$$

Sufficiency is straightforward to prove by using Roy's Identity. Applying Roy's Identity to (32) and rearranging yields:

(A52)

$$X_{MLj} = \sum_{i=1}^{N} x_{MLj}^{i} = \sum_{i=1}^{N} \frac{-\frac{\partial v^i}{\partial p_j}}{\frac{\partial v^i}{\partial y^i}} = \sum_{i=1}^{N} \int_{\gamma^i} \left(\frac{-\frac{\partial A^i}{\partial LS_{ML}^i(\gamma^i)}}{\overline{B}} \frac{\exp(\alpha_j^i(\gamma^i) - \beta^i(\gamma^i) p_j)}{\sum_{k=1}^{M} \exp(\alpha_k^i(\gamma^i) - \beta^i(\gamma^i) p_k)} \right) f(\gamma^i) d\gamma^i ,$$

where:

(A53) $$c_{ML}^i(LS_{ML}^i(\gamma^i), \gamma^i) = \frac{-\frac{\partial A^i}{\partial LS_{ML}^i(\gamma^i)}}{\overline{B}} ,$$

(A54) $$s_{MLj}^i(p_1, ..., p_M, \gamma^i) = \frac{\exp(\alpha_j^i(\gamma^i) - \beta^i(\gamma^i) p_j)}{\sum_{k=1}^{M} \exp(\alpha_k^i(\gamma^i) - \beta^i(\gamma^i) p_k)} = \frac{\partial LS_{ML}^i(\gamma^i)}{\partial p_j} .$$

Eq. (A53) is positive from (A50). From (A52) to (A54), we derive (31).

Appendix 5 Proof of Proposition 6

From (32), the expenditure function of the representative consumer is:

(A55) $$E = \frac{V - \sum_{i=1}^{N} \int_{\gamma^i} \left(A^i(LS_{ML}^i(\gamma^i), \gamma^i) \right) f(\gamma^i) d\gamma^i}{\overline{B}} .$$

From (A55), we obtain the Hicksian market demand function for good j as follows:

(A56) $$H_{MLj} = \frac{\partial E}{\partial p_j} = \frac{-\sum_{i=1}^{N} \int_{\gamma^i} \left(\frac{\partial A^i}{\partial LS_{ML}^i(\gamma^i)} \frac{\partial LS_{ML}^i(\gamma^i)}{\partial p_j} \right) f(\gamma^i) d\gamma^i}{\overline{B}} .$$

Because the Hicksian and Marshallian demand functions are the same when the indirect utility function is quasi-linear, from (34), (A55), and (A56), we obtain:

$$EV = \int_{p_j^{iW}}^{p_j^{iWO}} H_{MLj} dp_j$$

$$= \int_{p_j^{iW}}^{p_j^{iWO}} \left(\frac{-\sum_{i=1}^{N} \int_{\gamma^i} \left(\frac{\partial A^i}{\partial LS_{ML}^i(\gamma^i)} \frac{\partial LS_{ML}^i(\gamma^i)}{\partial p_j} \right) f(\gamma^i) d\gamma^i}{\overline{B}} \right) dp_j$$

(A57) $$= \int_{p_j^{iW}}^{p_j^{iWO}} \left(\sum_{i=1}^{N} \int_{\gamma^i} \left(c_{ML}^i (LS_{ML}^i(\gamma^i), \gamma^i) \frac{\partial LS_{ML}^i(\gamma^i)}{\partial p_j} \right) f(\gamma^i) d\gamma^i \right) dp_j$$

$$= \sum_{i=1}^{N} \int_{\gamma^i} \left(\int_{p_j^{iW}}^{p_j^{iWO}} x_{MLj}^i (LS_{ML}^i(\gamma^i), \gamma^i) dp_j \right) f(\gamma^i) d\gamma^i$$

$$= \sum_{i=1}^{N} \int_{\gamma^i} \left(\int_{LS_{ML}^{iW}(\gamma^i)}^{LS_{ML}^{iWO}(\gamma^i)} c_{ML}^i (LS_{ML}^i(\gamma^i), \gamma^i) dLS_{ML}^i(\gamma) \right) f(\gamma^i) d\gamma^i .$$

Appendix 6 Multiple Groups in the Case of the Mixed Logit Model

Because the analysis of multiple groups in the mixed logit model is similar to that in the logit model described in Appendix 3, in this appendix, we simply state results.

When there are multiple groups, the market demand function that is consistent with the mixed logit model is:

(A58) $$X_{MLj} = \sum_{i=1}^{N} \int_{\gamma^i} c_{MLg}^i (p_1, ..., p_M, y^i, \gamma^i) s_{MLgj}^i (..., p_k, ..., y^i, \gamma^i) f(\gamma^i) d\gamma^i ,$$

where $c_{MLg}^i(p_1,..., p_M, y^i, \gamma^i)$ is consumer i's demand for group g and $s_{MLgj}^i(..., p_k,..., y^i, \gamma^i)$ is consumer i's logit-type choice probability for group g, which is:

(A59) $$s_{MLgj}^i (..., p_k, ..., y^i, \gamma^i) = \frac{\exp(u_j^i(p_j, y^i, \gamma^i))}{\sum_{k \in g} \exp(u_k^i(p_k, y^i, \gamma^i))} .$$

The necessary and sufficient condition for the market demand function for good j to have the form of (A58) is that the indirect utility function of the representative consumer be given by:

$$\text{(A60)} \qquad V = \sum_{i=1}^{N} \int_{\gamma^i} \left(A^i(LS_{ML1}^i(\gamma^i), ..., LS_{MLG}^i(\gamma^i), \gamma^i) \right) f(\gamma^i) d\gamma^i + \overline{B}Y ,$$

which satisfies:

$$\text{(A61)} \qquad LS_{MLg}^i(\gamma^i) \equiv -\frac{1}{\beta_g^i(\gamma^i)} \ln \sum_{k \in g} \exp(\alpha_k^i(\gamma^i) - \beta_g^i(\gamma^i) p_k) .$$

The market demand function for good j, (A58), satisfies:

$$\text{(A62)} \qquad c_{MLg}^i(LS_{ML1}^i(\gamma^i), ..., LS_{MLM}^i(\gamma^i), \gamma^i) = \frac{-\dfrac{\partial A^i}{LS_{MLg}^i(\gamma^i)}}{\overline{B}} > 0 ,$$

$$\text{(A63)} \qquad s_{MLgj}^i(..., p_k, ..., \gamma^i) = \frac{\exp(\alpha_j^i(\gamma^i) - \beta_g^i(\gamma^i) p_j)}{\sum_{k \in g} \exp(\alpha_k^i(\gamma^i) - \beta_g^i(\gamma^i) p_k)} .$$

The own-price elasticity of the market demand for good j is:

$$\text{(A64)} \qquad \frac{\partial X_{MLj}}{\partial p_j} \frac{p_j}{X_{MLj}} = \sum_{i=1}^{N} \int_{w^i} \left\{ \frac{c_{MLg}^i s_{MLgj}^i}{X_{MLj}} \left(\varphi_{gj}^i - \beta_g (1 - s_{MLgj}^i) p_j \right) \right\} f(\gamma^i) d\gamma^i ,$$

where $\varphi_{gj}^i = \dfrac{\partial c_{MLg}^i}{\partial p_j} \dfrac{p_j}{c_{MLg}^i}$ is the elasticity of consumer i's demand for group g with respect to the price of good j, p_j. When goods j and j' belong to the same group, the cross-price elasticity is:

(A65) $$\frac{\partial X_{MLj}}{\partial p_{j'}}\frac{p_{j'}}{X_{MLj}}=\sum_{i=1}^{N}\int_{w^i}\left\{\frac{c_{MLg}^{i}s_{MLgj}^{i}}{X_{MLj}}\left(\varphi_{gj'}^{i}+\beta_{g}s_{MLgj'}^{i}p_{j'}\right)\right\}f(\gamma^i)d\gamma^i$$

Otherwise, the cross-price elasticity is:

(A66) $$\frac{\partial X_{MLj}}{\partial p_{j'}}\frac{p_{j'}}{X_{MLj}}=\sum_{i=1}^{N}\int_{w^i}\left(\frac{c_{MLg}^{i}s_{MLgj}^{i}}{X_{MLj}}\varphi_{gj'}^{i}\right)f(\gamma^i)d\gamma^i\ .$$

Equivalent variation can be calculated from consumer i's demand for good j or from consumer i's demand for group g; that is:

(A67) $$\begin{aligned}EV&=\sum_{i=1}^{N}\int_{\gamma^i}\left(\int_{p^{iW}}^{p^{iWO}}x_{MLgj}^{i}(LS_{ML1}^{i}(\gamma^i),...,LS_{MLG}^{i}(\gamma^i),\gamma^i)dp_j\right)f(\gamma^i)d\gamma^i\\&=\sum_{i=1}^{N}\int_{\gamma^i}\left(\int_{LS_{MLg}^{iW}(\gamma^i)}^{LS_{MLg}^{iWO}(\gamma^i)}c_{MLg}^{i}(LS_{ML1}^{i}(\gamma^i),...,LS_{MLG}^{i}(\gamma^i),\gamma^i)dLS_{MLg}^{i}(\gamma^i)\right)f(\gamma^i)d\gamma^i.\end{aligned}$$

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