Modelling the Effects of Range Uncertainty on Electric Vehicle Users’ Charging Behaviour

by

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April, 2013

Word Count including 250 x (# tables and figures): 8022
Abstract

Electric vehicle range is perceived as uncertain i.e. very difficult to predict by inexperienced users and arguably also by experienced users when operating it under unfamiliar conditions. This perception of range affects electric vehicles adoption, but also usage patterns and charging patterns, as it requires devising range resource management strategies. The decision making process underlying range management is therefore characterised by uncertainty. We explore the effect of range uncertainty on a subset of the range management decision making process: that of charging choices. This type of choices poses an interesting challenge for conventional methods of modelling choice in risky situations, all of which depend, in one form or another, on notions of the decision maker being able to identify the set of possible outcomes of a given choice and assign a subjective probability or decision weight to each possible outcome. We model charging choices under range uncertainty making distributional assumptions about the range attribute and adopting and Integrated Random Utility Expected Utility Theory approach. We combine alternative risk attitude specifications, namely constant absolute risk aversion and constant relative risk aversions, with alternative expressions for the range attribute, i.e. as absolute range and as buffer with the respect of the prospective driving distance after charging. We find that the best performing model in terms of data fit for our sample (specified with constant absolute risk aversion and range buffer) leads to range buffer willingness to pay estimates that are very sensitive to the distributional assumptions taken. This suggests that the exploration of alternative approaches not relying on distributional assumptions for the uncertain attribute is desirable.
Introduction

Setting the context

The limited range of electric vehicles (EV) in combination with their long recharging time at standard recharging rate and the lack of a rapid charging infrastructure make them a limited mobility resource when compared to vehicles that can rely on conventional fuels. This characteristic affects drivers’ vehicle adoption choices and electric vehicle driver’s mobility and charging choices. While the first set of choices in the electric vehicle context has been extensively studied, the second is poorly understood. Electric vehicle use and charging choices underlie a series of practical issues related to electric vehicles deployment that need to be addressed, ranging from demand side management (DSM) of EV charging for smooth integration of electric vehicles into grid systems to development of new businesses delivering integrated energy and mobility services. Practical contexts in which a sound understanding and modelling ability of behaviours characterising electric vehicle use and charging is crucial include the following:

- Realistically modelling load profiles \(^1\) from EV charging, whereas the current approach of electric vehicle deployment impact assessment relies on a limited range of pre-defined charging behaviour scenarios (Koyanagi and Uriu, 1997, Axsen and Kurani, 2008, Kang and Recker, 2009, Mullan et al., 2011);
- Assessing of price-based DSM strategies for electric vehicles load control;
- Designing services of future businesses such as Charging Service Providers or Electric Vehicles Aggregators, i.e. entities offering charging services to member electric vehicles, while lowering the aggregated cost to the grid of charging operations (Sundstrom and Binding, 2011, Bessa and Matos, 2012);
- Designing vehicle to grid concepts that allow electric vehicles–based grid operations enhancement, without conflicts with electric vehicle users needs.

These examples, along with the necessity to further deepen the understanding of adoption behaviour to model and propose policies to boost market uptake, demonstrate how unravelling the way the central issue of range perception affect all electric vehicle-related choices is timely and sensible. From recent research it seems to emerge that a key aspect of how the attribute range is processed in relevant choices is its perceived (and to a certain extent inherent) uncertainty. The brief review below presents how this finding has emerged; and provides an argument making worthwhile considering the effect of uncertainty when modelling charging behaviour.

The perception of range: current understanding

Vehicle type choice stated preference (SP) studies on samples drawn from the general drivers, typically with no experience with electric vehicles, have highlighted a large sensitivity for the range attribute (Beggs et al., 1981, Brownstone et al., 1996, Train and Hudson, 2000). However, as the stated adaptation games carried out by Turrentine et al. (1992) have revealed, drivers appear to be able to cope with the constraints of electric mobility, with minimal or no modification of their travel patterns. This evidence lead to the conclusion that the sensitivity for the range attribute as elicited in (SP) studies was inflated by lack of hands-on everyday experience with electric vehicles and that the apparent disutility of “limited mobility resources” was in fact reflecting more an aversion towards an unfamiliar technology (electric cars) assessed against a very familiar one (conventional cars), (Kurani et al., 1996). This position is however partially challenged by findings from post-electric vehicle trials interviews. Field research indicates that, although electric vehicle users easily adapt to electric vehicle use, they find that adequate ranges for daily use should be higher. Everett et al. (2011) found that trial participants, despite being able to adapt to EV use and despite a drop in stated concern about not reaching their destination explained by the increase in understanding of vehicle performances and of vehicle response to driving styles, still consider that an adequate range for their daily needs

\(^{1}\) It is in fact recognise that individual behaviour may have strong effects on charging patterns, see for example Lemoine et al. (2008)
should be higher. This contradiction may be symptomatic of a mismatch between the observed regular travel patterns to which participants have adapted and which cause less range-related concern than prior to the trial experience, and the perceived full spectrum of daily patterns which may include journeys for which the range is still of concern unless it is extended. Moreover, vehicle use data from trials, results of range games and in-depth questionnaires administered to trial participants revealed partial range use as compared to potential reference nominal ranges for their vehicles (Golob and Gould, 1998, Botsford and Szczepanek, 2009, Franke and Krems, 2013). Therefore “range issue” should not be simply reduced to a simple problem of lack of information and judgment instruments available to potential car buyers to correctly assess the potential of electric cars to satisfy their travel needs, when making purchase decisions. Management of the range resource is fact affecting electric vehicle drivers’ everyday driving behaviour, travel and charging patterns. The question that we therefore need to address is about the effect perception of electric vehicle range on electric vehicle use behaviour, where with the word use we intend both driving and charging. Based on interviews about their “range experience” to participants of a 6-month trial, Franke and Krems (2013) assert that the underutilisation of the range resources observed in electric vehicle trials reflects the adoption of safety strategies (e.g. use of safety range buffers) in electric vehicle use intended to avoid to encounter stressful problematic situations, and they propose that the “avoidance of stress might characterise range experience more than the experience of stress itself”. Franke and colleagues hypothesise that the adaptation strategies devised by trials participant are related to range uncertainty: 10 out of the 36 electric vehicle drivers they interviewed revealed that range was dependent on factors they could not relate to the EV or themselves. In initial support to this hypothesis they found as significant predictor of comfortable range, individual ambiguity tolerance. Also Graham-Rowe et al. (2012) report that un-ability to predict range generate range anxiety. The analyses of electric vehicle use based on trials and other experiments mentioned above lead to the idea that the perceived unpredictability of the range attribute is particularly important in determining electric vehicle use behaviour as characterised by range resource management. Clearly managing range translates into travel choices, (destination, routes, time of travel), driving behaviour and use of heating/cooling, but also of course on charging choices, as charging is the procedure by which the available range is increased.

Paper structure

Because charging choices are particularly important for price-based charging DSM measures and charging service supply in which users need to trade cost of charging, battery level and charging operation duration, the authors deem worthwhile analysing the effect of range uncertainty in this type of choice context. Thus the remainder of this paper will solely focus on how range uncertainty affect charging choices. In particular the paper will provide an initial exploration of the application to this novel context modelling approaches for choices under uncertainty. The remainder of this paper is organised as it follows. Section two introduces the modelling methodology, where after a brief review of the theoretical issues around choice under uncertainty, the models adopted in this paper are formally expressed. Section three introduces the data set utilised for model estimation; empirical models’ specifications and estimation results are discussed in section 4. The conclusions are presented in the final section.

Modelling Methodology

Outline of the choice situation

We intend to model a charging choice situation in which the electric vehicle user is uncertain about the actual range his electric vehicle will deliver and is only aware of interval boundaries within which the actual range will fall, for a given battery level achieved during the charging operation. The range interval boundaries for a given battery are assumed to be known by the drivers because could be provided for instance by the charging equipment interface. Because the range achieved depends on variables such as driving style, use of heating or cooling, vehicle load and road gradient, choosing which battery level to depart with entails facing ambiguity, for inexperienced electric vehicle drivers who are unaware on the range distribution,
conditional on these variables. Experienced users may be able to predict the range over a set of familiar journeys, but they may face uncertainty occasionally when they decide to use their electric car for unfamiliar journeys or driving conditions.

Underpinnings of models for choices under uncertainty

The type of choice situation described above is best termed as **uncertain** or **ambiguous**, because the outcome probabilities of the decision are unknown to the decision maker, whereas choice situations in which the probabilities of the potential outcomes are known to the decision maker are called **risky**.

Choice modelling under risk or uncertainty has been dominated for over six decades by expected utility theory (EUT) and its extensions that take into accounts well known observed deviations from the axioms underlying the original theory. de Palma et al. (2008) define as key deviations from expected utility theory: the Allais paradox (Allais, 1953), the preference reversal paradox and the Ellsberg paradox (Ellsberg, 1961). The first consists in an experimental demonstration that decision makers in lottery experiments are found not indifferent to prospects having the same expected utility, in clear contradiction with EUT. This type of evidence has lead to approaches that introduce weighting functions to the outcome probability to take into account for risk aversion (Kahneman and Tversky, 1979, Quiggin, 1982, Schmeidler, 1989). Note that EUT risk aversion is accounted for only using nonlinear utility functions (a concave utility function implies risk aversion as the expected utility of a prospect is lower than utility of the expected value of the prospect). De Palma et al. (2008) argue that when one is interested in measuring the utility parameters, i.e. estimating the utility parameters in empirical settings, neglecting the effect of probability weights leads to the overestimation of the concavity of the probabilities.

The second consists in changes in preference when the question is reframed, without relevant changes to the choice problem. One example of this type of framing problem is represented by the observed asymmetry in the perception of gains and losses, where the latter are in many domains weighted more than the former by people undertaking lotteries. Tversky and Kahneman (1992) address this type of behaviour with their Cumulative Prospect Theory.

The third is an experimental demonstration of the fact that problems under uncertainty are substantially different from problem under risk, whereas in economic literature uncertainty problems have often been reduce to risk problem introducing subjective probabilities (decision makers facing uncertain choice situation make use a subjective distribution, that can be theoretically inferred by their observed preferences).

Despite this evolution of the theoretical frameworks for modelling choice behaviour under uncertainty the application of these developments in practical context is still not widespread. In travel choice literature, which arguably constitutes part of the context in which the choices analysed in this paper belong to, only choices under risk have been traditionally studied. This was traditionally done integrating random utility theory (RUM) and EUT, with linear utility specifications, thus implying risk neutrality (Hensher et al., 2011). However in the last decade a considerable effort has been put to include characterisation of the risk attitude towards travel uncertainties.

de Palma and Picard (2005) neatly integrate RUM and EUT and use a nonlinear specification of the utility function that allows the estimation of a risk attitude parameter in the context of route choice under travel time variability. This framework is later applied on a more complex specification of the outcome value by Liu and Polak (2007). The context is still that of travel time variability, but the outcome value is specified not only in term of travel time but also of schedule delays and models estimated on data collected by Bates et al. (2001). The specification in term of schedule delays induces nonlinearity in the value of the outcome that in classical EUT is usually assumed as linear (whereas the utility of the value is specified nonlinearly to model the risk attitude). Adopting a nonlinear specification for the utility they disentangle nonlinear valuation for the attributes and risk aversion. In a later work data Polak et al. (2008) utilising the same approach and data but specifying a more flexible model characterise observed and unobserved risk attitude heterogeneity in context of travel time variability. More recently Hensher et al. (2011), also in the context of travel time variability, extend the framework, incorporating probability weighting functions, to account the Allais paradox effects. In specifying the nonlinear utility function they adopt and attribute-specific transformation, distorting only the attribute characterised by random variability. Hu et al. (2012) compare EUT and non-EUT models (i.e.
models in which probability weights are applied) for travellers’ risky choices using revealed preference data. They find inter alia that using RP data, the difference of fit between models is insignificant, due probably to the fact that that RP data cannot provide enough variation in travel time and induces risky outcomes. Their findings question the necessity to use non-EUT models instead of EUT models and require further exploration of non-EUT models within a revealed preference context before they can be applied reliably to modelling risky choices in the real world.

Empirical work in practical contexts in which choice under uncertainty is explicitly addressed (i.e. not treated as risk) is rare in general, and to the authors’ knowledge virtually absent in travel choice literature.

For our analyses we adopt the RUM-EUT approach, making assumptions about the underlying distribution for the random variable range. Notwithstanding the main limitations of this approach in the light of the brief literature review above (i.e. reducing uncertainty to risk and relying on assumptions for the perceived distribution of the range variable), we deem this approach sufficient to highlight the potential effect of risk aversion on the valuation of the range variable.

RUM-EUT models

Liu and Polak (2007) define the expected utility of a (risky) prospect $s_{nm}$ that a decision maker $n$ can choose within an $I_n$-dimensional choice set $C_n$ as

$$ U_n(s_{nm}) = \sum_{s \in s_{nm}} g(v_s, \alpha_n) p_s + \varepsilon_{nm} \quad (1) $$

Where $s$ is one of the possible outcomes of the prospect $s_{nm}$, $v_s$ is its value function typically expressed as a function of the attribute of $s$, $p_s$ the outcome probability. The function $g$ is a nonlinear transformation of the value function whose concavity indicates the risk attitude of the decision maker and $\alpha_n$ is a parameter determining the convexity of $g_n$ (that in general varies across individuals). The term $\varepsilon_{nm}$ is an observation error pragmatically applied to the complete prospect instead of the individual outcomes. With this formulation under standard assumptions regarding the error term distribution, standard models for discrete choice analyses can be derived. Typical expressions for $g$ are given in Table 1.

<table>
<thead>
<tr>
<th>Expression for $g$</th>
<th>Used by</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant absolute risk aversion (CARA)</td>
<td>$g(x, \alpha) = \frac{1 - e^{-\alpha x}}{\alpha}$</td>
</tr>
<tr>
<td>Constant relative risk aversion (CRRA)</td>
<td>$g(x, \alpha) = \frac{x^{1-\alpha}}{1-\alpha}$</td>
</tr>
</tbody>
</table>

An alternative formulation to that presented in equation 1 is an attribute specific one. In such a formulation the expectation operator is applied only to the attribute(s) characterised by random variability (Hensher et al., 2011). Therefore if we have a single random attribute with which occurs with levels $r_i$ occurring with $p_i$ probability we have

$$ U_{ni} = \beta_{ni} x_{ni} + \sum_{i \in s_{ni}} g(\beta_i r_i, \alpha_n) p_i + \varepsilon_{ni} \quad (2) $$

where $\beta_{ni}$ is its marginal value and $\beta_n$ is the vector of the marginal utilities of the non-random attributes $x_{ni}$ (under a linear-in-parameters specification).

Data description

Stated preference data

The stated preference dataset used for model estimation is part of a survey tool developed to study charging behaviour of electric car users in advanced "smart" charging scenarios in which the cost of the charging operation varies depending on the charging service. The stated choice exercise generating the data for the analyses in this paper is constituted by 12 binary choice
situations in which respondents choose "smart charger settings", to configure a charging operation taking place while their electric vehicle is parked at home, before undertaking a planned return journey (a home based tour in travel demand literature terminology). Respondent are familiar the tour characteristics, timings, purpose and distance as this was extracted from a one day travel diary they filled in at an earlier stage of the survey.

The charging alternatives are defined in terms of target battery level (i.e. the battery level at the end of the charging operation) duration of the charging operation and cost. These are the three design variables for the choice task. The battery level before charging and the start charging time are fixed across alternatives as well as across choice situations. While the former is also fixed across individuals, the latter is individual-specific. The choice attributes levels are the following:

- 4 levels of the target battery level. These are comprised between the minimum energy to charge to make the tour feasible (excluded) and the maximum energy that can be charged into the battery, given the battery state of charge before charging, the battery capacity (24kWh) and the maximum charging power (7.2 kW).
- 4 levels of charging operation duration. These are comprised between a minimum given by the time to charge the minimum energy to make the tour feasible at the maximum charging power (excluded) and the respondent’s vehicle dwell time at home before departure as observed from respondents’ one-day car diaries.
- Cost levels are obtained by three unit price levels multiplied by the amount of energy charged in the corresponding alternative.

The experimental design is based on a respondent-specific efficient design approach. This approach is similar to one proposed by Rose et al. (2008) for design of choice experiments in presence of a reference alternative. For each respondent a column based swapping algorithm runs for few iterations while the stated choice exercise page is loading, in search for designs decreasing the D-error calculated from the respondent-specific asymptotic variance-covariance matrix given priors estimates obtained from a pilot of the survey. Alternatives, in which the energy and charging duration levels imply a charging power higher than the maximum, are excluded. The number of swaps is such that the algorithm does not run for more than a couple minutes to avoid a long pause before the stated choice task page is loaded.

Although the alternatives are generated only based on the three design variables described above, these are presented to respondents in an extended form to ensure enough clarity in their description (Figure 1). Amongst the additional information provided to the respondents a driving range interval typically achievable with the given target battery level is shown. Its extremes are obtained multiplying the target battery levels by 2 fixed efficiency values corresponding to the least efficient condition and the most efficient condition. This information was originally provided to remind respondents that for a given battery level depending on their driving style, usage of heating or air conditioner and road/traffic condition range may vary. In practice describing the range in such fashion highlights the uncertainty that the alternatives charging choice settings bear regarding the achievable range.

Although the choice exercise presented was not designed principally to elicit the risk attitude of the respondents, it presents choice situations in which the distance of the tour respondents are imagining to undertake after charging, falls above the low extreme of the range interval (though always below the high extreme): the risk of “remaining stranded” may affect the choice in these situations, depending on the risk attitude of the respondents.

Additionally, at the end of the questionnaire, respondents face a question intended to obtain their expectation regarding effective driving distance they could get with a fully charged battery, assuming that they range they usually get is between 60 and 100 miles. This question was specifically phrased as follows: “Suppose that on a full battery you usually get between 60 and 100 miles before it runs out. How far would you expect to be able to drive after the next full charge?” Respondents can choose within a range between 0 and 100 miles. The purpose of this question was to elicit from the respondents a piece of information about the potential subjective distribution they might have in mind when presented the range attribute in the form of an

2 Respondents are car owning drivers, not EV owners. They are asked to imagine that an EV is substituting their car. The EV characteristics are described to them and at an earlier stage of the survey they familiarise with them undertaking a stated adaptation task of their car diary to accommodate its use and charging.
interval. We thus use the answer to this question is used to inform one of the distributional assumptions of the uncertain range attribute as it is described in the following section.

**Figure 1** Example of charging choice task

Dataset descriptive statistics

From the stated preference data described only choice situations in which at least one alternative has a non-zero risk of “remaining stranded” are used for model estimation. The number of observations is 755 from 88 UK car drivers, who are asked to choose between two alternative charging settings before undertaking a tour with distances that are fixed across choice situation but vary across respondents between 30 and 80 miles. The distributions of these distances and of vehicle dwell time at home before departure used to define the charging duration levels are reported in Figure 2, together with the answer to expected range question described above. Out of the 755 observation 489 correspond to choice situations in when both alternatives are “risky”, in the sense that there is a non-null probability for the event “remaining stranded”.

**Figure 2** Distributions of driving distances after changing, vehicle dwell time before departure and stated expected range with full charge

**Specifications of empirical models and estimation results**

Empirical models specifications

With the CRRA transformation only using this attribute specific formulation (equation 2) allows the estimation of separate attribute specific parameters that allow a straightforward calculation of the willingness to pay (Hensher et al., 2011). To allow a direct comparison between CRRA and CARA we opted to present here only results relative to attribute specific formulations. Therefore the utility for the charging setting alternative can be expressed using CARA as:
\[ U_{ni} = \beta_0 + \beta_1 C + \beta_2 T + \sum_{s \in \mathbb{S}_n} \frac{1 - \exp(-\alpha \beta_3 R_s)}{\alpha} p_s + \varepsilon_{ni} \quad (7) \]

where \( C \) is the charging cost, \( T \) the charging time and \( R \) the range. In the case of the CRRA, instead
\[ U_{ni} = \beta_0 + \beta_1 C + \beta_2 T + \gamma \sum_{s \in \mathbb{S}_n} (R_s)^{1-\alpha} p_s + \varepsilon_{ni} \quad (8) \]

In both cases extreme value type I distribution for the error term is assumed, leading to logit models.

We also present estimation results in which instead of the range attribute specified \textit{as is}, it is specified via the range buffer \( B_s \) with the respect to the distance of the tour that needs to be undertaken after charging, where \( B_s \) is equal to \( \max(R_s - \text{distance}, 0) \).

### Range distribution assumptions

The range interval corresponding to a given target battery level is split into three equal parts (subintervals). The midpoints of each subinterval are used as values for \( R_s \) (i.e. mass points of a discrete distribution) each assumed to occur with probability \( p_s \), with \( s = 1,2,3 \).

We tested two distributional assumptions for the range variable: a uniform distribution and a distribution based on respondents’ stated expected range for the full charge (\( ERFC_n \)). In this second case the distribution is chosen triangular with support equal to the range interval for the target battery level of the alternative and with mode given by
\[ M_{in} = ERFC_n \frac{SOC_{in}}{SOC^{full}} \quad (9) \]

where \( SOC_{in} \) is the target battery level (or State Of Charge) for alternative \( i \) and respondent \( n \) (in kWh), \( SOC^{full} \) the electric car battery capacity (24 kWh in the choice exercise for all respondents).

For respondents choosing an \( ERFC_n \) value equal to one of the extreme of the full battery range interval (i.e. 60 or 100 miles, see the formulation of the question about stated expected range for the full charge in the previous section), the triangular distribution is reduced to a truncated triangular distribution with support from the \( M_{in} \) to the high extreme of full charge range interval if \( ERFC_n = 60 \) miles, or from the low extreme of full charge range to the \( M_{in} \) if \( ERFC_n = 100 \) miles. The distributions are then made discrete by concentrating on the mass points \( R_s \) the distribution area comprised within the corresponding three subintervals. As it is defined this second distribution is respondent specific as it is defined based on the individual specific variable \( ERFC_n \). Thus hereafter we will refer to this distributional assumption as individual-specific triangular (IST).

The probabilities for the range occurrences \( R_s \) for the distributional assumptions outlined above are summarised in Table 2.

### Table 2 Distributional assumption for range occurrences given a target battery level

<table>
<thead>
<tr>
<th>( R_s )</th>
<th>Uniform</th>
<th>IST</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 67 \text{miles} \times SOC_{in}/SOC^{full} )</td>
<td>( p_1 )</td>
<td>33%</td>
</tr>
<tr>
<td>( 80 \text{miles} \times SOC_{in}/SOC^{full} )</td>
<td>( p_2 )</td>
<td>33%</td>
</tr>
<tr>
<td>( 94 \text{miles} \times SOC_{in}/SOC^{full} )</td>
<td>( p_3 )</td>
<td>33%</td>
</tr>
</tbody>
</table>

(*) \( F \) is the cdf of the triangular distribution with support 60-100 and mode \( ERFC_n \). 

### Estimation results discussion

We present here the estimation results we obtained. The estimation of all models was performed using BIOGEME 2.0, (Bierlaire, 2003, Bierlaire, 2009).

Table 3 reports the multinomial logit parameter estimates of the 12 specifications corresponding to the combination of: 3 RUM-EUT formulations (linear utility specification, attribute-specific CARA transformation of the range value function and attribute-specific CRRA transformation of the range value function), 2 specifications in which the range attributes enter the utility function (range as is and range buffer) and finally 2 distributional assumptions, that were described earlier in this section (uniform and IST). Hereafter we will adopt the following taxonomy to identify the various model specifications:
The suffixes Lin-, CARA- and CRRA- indicate the models respectively featuring a linear utility, a CARA nonlinear transformation of the value function and a CRRA nonlinear transformation of the value function;

The suffixes R-, B- indicate the models in which range is specified as is or in terms of buffer;

Combinations of the two specifications above are expressed by the following self explanatory suffixes: Lin/R-, Lin/B-, CARA/R-, CARA/B-, CRRA/R- and CRRA/B-.

The range distribution assumptions will be explicitly mentioned case by case.

We observe that for all estimated models, cost, charging duration and range parameters take the expected sign indicating a disutility for high cost and long charging times and a positive utility for high range values.

The risk attitude parameter $\alpha$ takes different signs depending on how the range is specified, making it more problematic to draw a general conclusion about the average attitude towards risk of our sample of car drivers when making charging choices. In fact we observe that in all B-specifications the sign of $\alpha$ and its magnitude suggests risk aversion. In particular the positive value of $\alpha$ in the CARA/B-specifications makes the g function in Table 1 concave and so do the positive and smaller than one $\alpha$ values in the CARA/B-specifications. However when the range enters the utility function as is one could reach the opposite conclusions (in the R-specifications), at least for the corresponding CARRA/R-specification where $\alpha$ is statistically significant. Such a conclusion would be less tenable if one considered the CARA/R-models where both the range parameter and $\alpha$ are not statistically significant. In general however purely based on model fit considerations one may argue that because the B-specifications have a better fit of the corresponding R-specifications regardless of the nonlinear transformation of the value function applied, one may be more confident on the conclusion they suggest: a general risk aversion for the sample in evaluating the uncertain range attribute, at least in the hypothetical setting of the stated choice exercise.

Concerning the distributional assumptions, the uniform distribution has a better fit of the data for the low performing model specifications (Lin-specifications and R-specifications), whereas the IST distribution outperforms in terms of fit the uniform distribution for the best fitting models (the CARA/B-specification and the CRRA/B-specification). The overall best fit is achieved by the CARA/B-specification under the IST range distribution assumption. The improvement in goodness of fit of the two best performing specifications when using distributional assumptions based on partial information regarding the potential underlying subjective distribution of the uncertain attribute suggests that is worth pursuing ways to try eliciting this type of information and using it for the specification of choice models with uncertain attributes.

Willingness to pay

In addition to the model estimates we present in Table 4 willingness to pay (WTP) estimates for the range buffer, calculated at the average probability levels and average buffer levels. In general the willingness to pay is calculated as the ratio between the marginal utility of an attribute and the marginal utility with respect to the cost coefficient. In the present case it is sensible to evaluate the marginal utility of the expected range buffer, which is problematic unless it is assumed that the Bs vary all the same amount. We adopt this assumption suggested by Polak et al. (2008) and implied by Hensher et al. (2011), which leads to the following expressions for the WTP:

$$WTP_b = \frac{\beta_b}{\beta_c}$$ (10)

$$WTP_b = \frac{\beta_b}{\beta_c} \sum_{s \in u} p_s e^{-\alpha B_s}$$ (11)

$$WTP_b = \frac{\beta_b}{\beta_c} \sum_{s \in u} p_s B_s^{-\alpha}$$ (12)

The first one is for the Lin/B-specification, the second one for the CARA/B-specification and the third one for the CRRA/B-specification. The best fitting model (CARA/B with IST distribution assumption) gives a WTP (£0.17/mile) that is close to the estimates provided by the Lin/B-
models. Instead the CRRA/B-estimates tend to give lower WTP estimates: £0.11/mile under the assumption of uniform distribution and £0.12/mile under the triangular distribution assumption. Note that there is a great variation in the WTP estimates across distribution assumptions when using CARA: assuming a uniform distribution the WTP estimate decreases of the 47%.

This high sensitivity suggests that large errors could be made in the estimation of the value of uncertain attribute depending on the adopted distribution. Therefore, alternative approaches that are less reliant on distributional assumptions and make more use of indicators partially revealing the potentially underlying subjective distributions are desirable. Such indicators should be carefully designed allowing the estimation of models for the latent characteristics of the subjective distributions based on individual characteristics.

Conclusions

This paper has presented an application field of choice modelling still very little explored, related to electric vehicle charging. Due to the inherent physical and perceived uncertainties regarding the electric vehicle range available for a given amount of energy recharged, charging choices are a particularly interesting context where to apply and test choice modelling framework that allow capturing the effect of an uncertain attribute on individual decisions. But the application of complex models accounting for range uncertainty on charging choices should not be viewed as a pure academic exercise because a whole range of problems and business interests are developing around charging behaviour of electric vehicle users. Thus understanding and modelling charging behaviour, capturing the effects of the most salient characteristics of the choice context that defines it has practical interest and is timely.

This paper focused specifically on modelling charging choices under range uncertainty. Acknowledging that uncertainty influences differently decision making from risk, we treated the choice problem, as it is often done in practice, as one under risk making assumptions regarding the range distributions achieved with a battery charged a given level. We compared different models specification explicitly accounting for risk aversion, via nonlinear transformations of the outcome value (specifically CRRA and CARA). We also compared two alternative ways to express the range attribute in the utility functions: range as is, i.e. as absolute value, and in terms of buffer i.e. of residual range above the prospective driving distance. The best fits for both CARA and CRRA were obtained using the buffer specification. The estimates of the risk attitude parameters for these best fitting models suggest that sample tended to be risk avert in facing the hypothetical charging choices.

Despite the practical advantages of treating a choice problem characterised by uncertainty as one characterised by risk, we highlighted in which ways this approach is also problematic, due to the reliance on the assumptions regarding the uncertain attribute distributions. We showed in particular how the willingness to pay estimates obtained using CARA were particularly sensitive to the distributional assumption, leading to a variation of the 47% of the willingness to pay estimate for the range buffer.

This example calls for the exploration of alternative approaches usable in practice to model choice under uncertainty, i.e. when the decision maker is unaware of the distribution of an attribute characterising the available alternatives in a choice. The authors suggest one based on Integrated Choice Latent Variable Model, in which the expectation of an uncertain attribute is modelled as latent a variable. Such a model could be estimated if suitable indicators measuring this latent expectation were available.
### Table 3: Estimated parameters

**Range uniformly distributed within range interval**

<table>
<thead>
<tr>
<th>Name</th>
<th>Lin/R-model</th>
<th>CARA/R-model</th>
<th>CARRA/R-model</th>
<th>Lin/B-model</th>
<th>CARA/B-model</th>
<th>CARRA/B-model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alternative specific constant A</td>
<td>Value</td>
<td>Std err</td>
<td>t-test</td>
<td>Value</td>
<td>Std err</td>
<td>t-test</td>
</tr>
<tr>
<td>Adjusted rho</td>
<td>0</td>
<td>fixed</td>
<td>0</td>
<td>0</td>
<td>fixed</td>
<td>0</td>
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<tr>
<td>Alternative specific constant B</td>
<td>-0.0426</td>
<td>0.0818</td>
<td>-0.52</td>
<td>0.0366</td>
<td>0.0859</td>
<td>0.43</td>
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<tr>
<td>Null log-likelihood</td>
<td>0</td>
<td>fixed</td>
<td>0</td>
<td>0</td>
<td>fixed</td>
<td>0</td>
</tr>
<tr>
<td>Cost (10E)</td>
<td>-3.34</td>
<td>0.503</td>
<td>-6.63</td>
<td>-4.89</td>
<td>0.584</td>
<td>-8.36</td>
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<tr>
<td>Charging duration (1000 miles)</td>
<td>5.42</td>
<td>0.576</td>
<td>9.42</td>
<td>0.159</td>
<td>0.082</td>
<td>1.94</td>
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<tr>
<td>Rho-square</td>
<td>0.113</td>
<td>0.169</td>
<td>0.171</td>
<td>0.085</td>
<td>0.173</td>
<td>0.164</td>
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<tr>
<td>Adjusted rho-square</td>
<td>0.105</td>
<td>0.16</td>
<td>0.162</td>
<td>0.078</td>
<td>0.173</td>
<td>0.164</td>
</tr>
</tbody>
</table>

**Range triangularly distributed within range interval**

<table>
<thead>
<tr>
<th>Name</th>
<th>Lin/R-model</th>
<th>CARA/R-model</th>
<th>CARRA/R-model</th>
<th>Lin/B-model</th>
<th>CARA/B-model</th>
<th>CARRA/B-model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alternative specific constant A</td>
<td>Value</td>
<td>Std err</td>
<td>t-test</td>
<td>Value</td>
<td>Std err</td>
<td>t-test</td>
</tr>
<tr>
<td>Adjusted rho</td>
<td>0</td>
<td>fixed</td>
<td>0</td>
<td>0</td>
<td>fixed</td>
<td>0</td>
</tr>
<tr>
<td>Alternative specific constant B</td>
<td>-0.0492</td>
<td>0.0813</td>
<td>-0.61</td>
<td>-0.00678</td>
<td>0.0833</td>
<td>-0.08</td>
</tr>
<tr>
<td>Cost (10E)</td>
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<td>0.498</td>
<td>-6.36</td>
<td>-3.97</td>
<td>0.543</td>
<td>-7.32</td>
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<tr>
<td>Charging duration (1000 miles)</td>
<td>5.02</td>
<td>0.55</td>
<td>9.11</td>
<td>0.334</td>
<td>0.186</td>
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<tr>
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<td>0.131</td>
<td>0.132</td>
<td>0.08</td>
<td>0.216</td>
<td>0.194</td>
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<tr>
<td>Adjusted rho-square</td>
<td>0.098</td>
<td>0.122</td>
<td>0.123</td>
<td>0.073</td>
<td>0.206</td>
<td>0.184</td>
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</tbody>
</table>
Table 4 Willingness to pay estimates

<table>
<thead>
<tr>
<th>WTP for range buffer(*)</th>
<th>Linear utility</th>
<th>CARA</th>
<th>CRRA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uniform range distribution</td>
<td>1.8</td>
<td>0.9</td>
<td>1.1</td>
</tr>
<tr>
<td>Triangular/truncated triangular range distribution</td>
<td>1.7</td>
<td>1.7</td>
<td>1.2</td>
</tr>
</tbody>
</table>

(*) values in £10 per 100 miles, calculated at the average probability and buffer levels in the sample:

- \( p_1 = p_2 = p_3 = 33\% \) for uniform range distribution;
- \( p_1 = 28.5\%, p_2 = 27.2\% \) and \( p_3 = 44.3\% \) for triangular/truncated triangular range distribution;
- \( B_1 = 6\) miles, \( B_2 = 13\) miles and \( B_3 = 23\) miles.

References


Bierlaire, M. 2009. Estimation of discrete choice models with BIOGEME 1.8. Transport and Mobility Laboratory, EPFL.


