A Utility Theory for Logit Models with Repeated Choices

Abstract – Logit models with repeated choices are widely used in empirical applications, but connecting the model to a theory of utility-maximizing behavior has proved difficult. McFadden’s random utility maximization (RUM) provides a supporting theory for a single discrete choice, but not for the series of choices that characterizes consumer behavior in many circumstances. We show that a logit model with repeated choices can be viewed as a system of demand equations satisfying integrability conditions for consistency with utility maximization. The proposed approach permits the calculation of theoretically valid, income-adjusted welfare measures for this common form of discrete-choice model.

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I. Introduction

Logit models with repeated choices are widely used in empirical applications, but authors have not been able to fully connect the model to a theory of utility-maximizing behavior. While McFadden’s (1974) theory of random utility maximization (RUM) is generally invoked as the basis for demand and welfare formulas in any logit discrete-choice model, RUM theory appropriately applies only to the case of a single discrete choice. For example, RUM theory applies to a contingent valuation experiment in which an individual chooses once between two states of the world. It cannot be applied to a pattern of repeated choices, such as the selection of which restaurant to go to on each of numerous nights out in a year. The reason is that RUM theory associates any given choice with a particular indirect utility function, and multiple choices by the same individual imply multiple utility functions without any explicit connection to a single individual and a single preference ordering.

Repeated-choice logit is frequently used in the field of recreation demand to measure the impact of environmental changes on the value of outdoor recreation. The results are used to evaluate environmental policies, and to assess damages in natural resource damages litigation following oil spills and other incidents. On a given choice occasion an individual selects one of J alternatives, which include taking a trip to one of several recreation sites, and may include the decision not to take a trip. The problem is one of repeated choices because individuals often take numerous recreation trips in a year. The RUM expression typically applied to each choice occasion is

\[ U_{ij} = V(y_i - p_{ij}, x_i, z_i) + \varepsilon_{ij}. \]

(1)

Conditional indirect utilities \( U_{ij} \) describe individual \( i \)'s utility ranking over the \( J \) alternatives, including \( J-1 \) recreation sites and the “no-trip” option. \( V_{ij} \) is observable preferences (relating to income \( y_i \), prices \( p_{ij} \), site characteristics \( x_i \), and individual characteristics \( z_i \)) and \( \varepsilon_{ij} \) represents the influence of unobserved factors. For a single choice among recreation destinations and the option to do something else, \( U_{ij} \) is a complete utility ranking. However, \( U_{ij} \) does not provide a utility ranking over the set of options an individual faces in a year. For example, \( U_{ij} \) does not describe whether two trips to site A and three trips to site B, on the one hand, are preferred to six trips to site C, on the other.

There have been several proposals in the literature for a unified theoretical model of repeated choices. Hausman et al. (1995) used two-stage budgeting theory (Gorman 1959) to link the second-stage choice of a recreation site, specified using logit probabilities, with a first-stage demand function for the total number of trips to all sites. Several authors noted a critical mistake in the way the Hausman et al. model was derived (Smith 1997; Herriges et al. 1999). Similar strategies linking a repeated-choice model of site selection with a demand function for total trips were explored by Bockstael et al. (1987), Parsons and Kealy (1995), Feather et al. (1995), and Parsons et al. (2009). A second theoretical approach common in the literature assumes independence across an individual’s repeated choices (Morey et al. 1993; McFadden 1999; Herriges and Kling 1999; Shaw and Ozog 1999). By itself, each independent choice is consistent with random utility maximization. Demand and welfare associated with an individual’s repeated choices are calculated as the sum over RUM-based demand and welfare predictions for each independent choice.

This latter approach, using RUM theory with the assumption of independent choices, is almost universally adopted among researchers in the field of recreation demand. The
approach has also been criticized (McConnell 1995; Herriges et al. 1999; von Haefen et al. 2004) and some view it as an approximation (Smith and von Haefen 1997). To motivate the analysis in this article, two empirical flaws of this approach are noted here. First, the calculation of RUM-based welfare measures involves adjustments to income $y_i$ in equation (1) that either offset (compensating variation) or reproduce (equivalent variation) a change in utility caused by a change in the characteristics $x$ of the choice alternatives. Under the assumption of independent choices, the income adjustment in (1) is applied to only a single choice. However, income is defined for individuals not choices, and any adjustment to income must appropriately be applied to all choices an individual makes. The practice of ignoring the influence of income adjustments across an individual’s repeated choices can be expected to cause a downward bias in the magnitude of income effects when computing the value of changes in $x$. This downward bias is evident in results presented later.

The second empirical flaw involves the standard practice of allocating an individual’s annual income across repeated choices. In other words income $y_i$ in (1) is “per-period” income, where the number of periods is equal to the number of choices an individual makes in a year, and per-period income is equal to annual income divided by the number of periods. This practice may be an attempt to circumvent the first problem, noted above. Unfortunately, the number of choices in a year is either endogenous (if the set of modeled choices is an individual’s total number of recreation trips) or arbitrary (if the “no trip” option is included, and the set of modeled choices is based on days, weeks, or some other convenient period of analysis). This means that per-period income has no consistent relationship to annual income, but is instead directly dependent on the number of choices used to specify the model. The empirical effect is that the coefficient on income, and the magnitude of income effects, is arbitrarily small or large depending on the researcher’s selection of an applied specification and the resulting scale of the income variable. This result is also shown later in the empirical analysis.

In this article we present an alternative utility theory for logit models of repeated choices based on integrability conditions derived for systems of demand equations. Demand-system theory relies on a representative-agent model, using the assumption that differences in behavior not explained by observed individual characteristics can be represented as average behavior. This is a departure from the assumption of heterogeneity in unobserved preferences typical in logit discrete-choice analysis. However, it is analogous to the Hausman et al. (1995) approach to utility-consistency based on two-stage budgeting, which is also a representative-agent model. Our approach has further similarities to a series of studies establishing equivalence between representative-agent models and RUM-based choices aggregated over numerous individuals. For example, Anderson et al. (1987, 1992) show that demand and welfare for a representative agent with a CES utility function can in certain cases be identical to total demand and welfare for a population of individuals, each choosing among RUM-specified discrete alternatives. In most of these studies, individuals each make a single RUM-based choice (Anderson et al. 1987, 1992; Verboven 1996) rather than a series of repeated choices. One attempt was made to extend these methods to account for repeated choices (Smith and von Haefen 1997), but the assumptions of per-period income and independence across repeated choices were retained, leaving unresolved the problems noted above.

In our proposed demand-system approach, integrability conditions derived for incomplete demand systems (LaFrance and Hanemann 1989; LaFrance 1990) are applied to the demand equations implicit in repeated-logit models. Consider a nested logit formulation for recreation trips. In the familiar RUM context, indirect utility would be

$$U_{ij} = V_{ij} + \varepsilon_{ij} = \beta_p p_{ij} + \beta_x x_j + \varepsilon_{ij}$$
where $U_j$ ($j > 0$) is the utility of a visit to a recreation site and $U_{i0}$ is the utility of activities other than recreation.\(^1\) If $K$ choice occasions are specified, the demand for site $j$ by individual $i$ is the familiar nested-logit expression

$$q_{ij} = K \frac{\left( \sum_j e^{V_{ij}} \right)^{1/2} e^{V_{ij}}}{e^{V_{i0}} + \left( \sum_j e^{V_{ij}} \right)^{1/2} \sum_j e^{V_{ij}}}.$$  (3)

$K$ may represent days or weeks, but it could also be any number large enough to account for the maximum number of trips an individual takes in a year. In a RUM model, the terms following $K$ on the right-hand side represent the probability of taking a trip, and the probability of choosing site $j$ conditional on taking a trip, respectively. The scale parameter $s$ measures the degree of substitution between choice alternatives. While the utility of choosing activities other than recreation plays an explicit role in the RUM context, in the demand-system context the term $V_i$ in (3) becomes a shifter in the demand equations for the $J$-1 recreation sites. The demand system in (3) is thus “incomplete” because no demand equation is specified for goods other than recreation trips.

The next section describes integrability conditions for utility consistency of an incomplete demand system, and illustrates how (3) satisfies them. Empirical results include a comparison of previous welfare measures with welfare measures based on the area under income-compensated, repeated-logit demand functions. Our conclusion is that model specifications widely used for repeated-choice problems can be supported by a valid utility theory, but the theory requires a revision in the way utility-consistent welfare measures EV and CV should be calculated.

II. Integrability and Repeated-Choice Logit

It is a basic result of consumer theory that an individual’s Marshallian demand equations for multiple goods are consistent with rational maximizing behavior if and only if the Slutsky matrix of substitution terms associated with the demand equations is both symmetric and negative semi-definite. These conditions for theoretical validity are known as “integrability conditions”, named for the integration problem that can be solved to recover expenditure and utility functions from observed Marshallian demands. Unlike RUM theory, which explicitly specifies a preference ordering for choice alternatives, integrability conditions establish that a rational preference ordering exists that is consistent with a specific set of demands, even if the preference ordering cannot be made explicit. This demand-system approach was first applied to recreation trips by Burt and Brewer (1971), and the modifications to integrability conditions appropriate for an incomplete system of demands were developed by Epstein (1982) and LaFrance and Hanemann (1989). Since then numerous articles have used integrability conditions to develop models of recreation demand consistent with utility theory (von Haefen 2002; Moeltner 2003; Herriges et al. 2008). Repeated logit is more widely used than any of these models, presumably because it is thought to perform better empirically.

It is straightforward to show that the demand equations in (3), with $V_{ij}$ and $V_{i0}$ specified by (2), are consistent with symmetry of the Slutsky substitution matrix. Slutsky symmetry is the key condition that ensures the validity of the structure of a demand specification. Negative semidefiniteness may hold only locally for specific parameter values,
and will be tested empirically in the applied analysis below. Slutsky symmetry holds if, for any pair of goods (or recreation sites) \( l \) and \( m \),

\[
\frac{\partial q_l}{\partial p_m} + \frac{\partial q_l}{\partial y} q_m = \frac{\partial q_m}{\partial p_l} + \frac{\partial q_m}{\partial y} q_l. \tag{4}
\]

Each term on the left-hand side of (4) is matched by an analogous term on the right-hand side, with the subscripts \( l \) and \( m \) reversed. Writing out the terms on the left-hand side of (4) based on the specification in (2) and (3), we obtain

\[
\frac{\partial q_l}{\partial p_m} = e^{sv_l e^{sv_m K}} \frac{-s\beta e^{(\sum_j e^{sv_j})\frac{1}{2}}}{\left(\sum_j e^{sv_j}\right)^2 \left(\sum_j e^{sv_j}\right)^{\frac{1}{2}} + e^{v_0}} \left[ 1 - \frac{e^{v_0}}{s\left(\sum_j e^{sv_j}\right)^{\frac{1}{2}} + e^{v_0}} \right], \tag{5}
\]

and

\[
\frac{\partial q_l}{\partial y} q_m = e^{sv_l e^{sv_m K^2}} \frac{-s\beta e^{(\sum_j e^{sv_j})\frac{3}{2}}}{\left(\sum_j e^{sv_j}\right)^3 \left(\sum_j e^{sv_j}\right)^{\frac{1}{2}} + e^{v_0}} \left(\sum_j e^{sv_j}\right)^2.
\]

Reversing the subscripts \( l \) and \( m \) in (5) results in identical expressions, so the equality in (4) is satisfied.

Equation (3) is similar in form to demand-system specifications common in the literature. Specifically, many demand-system models are based on a semi-log expression for demand (LaFrance 1990). Equation (3) uses a similar exponential form, which becomes arbitrarily close to the semi-log demand function as the scalar \( K \) is increased (Hellerstein and Mendelsohn 1993; Parsons et al. 1999). However, in a semi-log demand system, integrability requires that Marshallian cross-price effects be fixed at zero (LaFrance and Hanemann 1989). Such unrealistic constraints are common in demand-system specifications, but are avoided in the system of equations in (3).

Many demand equations do not integrate back to a closed-form expenditure function \( e \) or indirect utility function \( V \) that can be used for welfare analysis. In other words, in many cases no solution exists to the partial differential equations \( \frac{\partial e(p,u)}{\partial p_j} = q_j(p,e(p,u)) \) or \( \frac{\partial V_j}{\partial y} = q_j(p,y) \) for all \( j \) given a specified form for \( q \). A search using Mathematica software indicated that no closed-form solution exists for the demand equations in (3).

However, welfare measures can instead be calculated based on changes in the area under income-compensated demand functions, using numerical integration methods developed by Vartia (1983) and Bullock and Minot (2006).

### III. The Demand for Trips to Mid-Atlantic Beaches

A model based on equation (3) was estimated using data on recreation trips to beaches in the Mid-Atlantic region of the United States. The data were collected in a mail survey, with 1,966 respondents reporting 3,910 beach trips during 2005. The survey was sent to a sample of residents in Delaware, New Jersey, New York, Ohio, Pennsylvania, Maryland, Virginia, West Virginia, and the District of Columbia. The choice alternatives represented in the data include all sandy beaches in New Jersey, Delaware, Maryland and Virginia, grouped into 66 aggregate sites.
The data report characteristics of survey respondents, such as age, sex, and income, as well as characteristics of the 66 sites, such as length, width, and level of development. A price variable for each choice alternative was constructed based partly on the monetary cost of traveling from an individual’s residence to each of the 66 sites. Monetary costs include the cost of gasoline, highway tolls, and parking fees. Prices also include the non-monetary cost of time spent driving, valued at one-third hourly household income. The full set of individual and beach characteristics are listed in Table 1.

Model estimation used maximum-likelihood methods standard in repeated-logit models, as described by Morey (1999). Specifically, data on trips were fit to a multinomial distribution in which expression (3), without the scalar $K$, enters the likelihood function once for each trip taken by individual $i$ to site $j$. The likelihood function also accounts for the number choice occasions when no trip is taken, which for a given individual equals $K$ less annual trips. For each occasion when no trip is taken, the term entering the likelihood function is one minus the sum of expression (3) across all sites (without the scalar $K$). Alternative estimation approaches more typical for demand-system specifications could involve fitting expression (3) to observed trips using a Poisson or negative-binomial distribution. Our estimation procedure is the most common approach for repeated logit, and is likely to produce results similar to those of a Poisson-based estimator (Hellerstein and Mendelsohn 1993; Parsons et al. 1999).

Two models were estimated, one with 100 choice occasions and one with 200 choice occasions ($K = 100$ and $K = 200$, respectively). In logit recreation models that include the “no trip” option, the decision of how many choice occasions to include in the model is essentially arbitrary. For example, some researchers use $K = 52$ to represent weeks in a year or $K = 365$ to represent days in a year. For model estimation, all that matters is that $K$ be at least as large as the greatest number of trips by any individual in the data.

Estimated model parameters are shown in Table 1. Most parameters are significant at the five-percent level and agree with prior expectations. Possible exceptions include the sign on suitability for surfing, which is negative and most likely indicates that many people who swim prefer to avoid beaches where surfing takes place. Development appears as a positive attribute, likely due to the availability of bars and restaurants as opposed to aesthetic considerations. The parameter associated with income is negative. Given the specification in (2) and (3), this means that beach trips are a normal good and that demand for trips increases with income. Using the estimated parameters in Table 1, the Slutsky substitution matrix given in (4) and (5) can be tested for negative semidefiniteness. A programming routine in GAUSS was used to calculate eigenvalues of the Slutsky matrix for each individual in the data. All eigenvalues for all individuals were nonpositive, so this second condition for utility consistency of the model is satisfied.

IV. Welfare Comparisons

The first section of this article described the application of RUM theory to a single discrete choice. It also described the conventional theoretical approach to repeated choices, which retains the RUM framework by assuming that each choice an individual makes can be treated as a single independent event. In this section we compare welfare results based on conventional RUM analysis with welfare results derived from a demand-system approach to repeated choices.
Table 2 shows several estimates of the welfare change resulting from a hypothetical decline in beach quality. The specific scenario is a decline in the width of all beaches in the study region, and the welfare changes presented are totals across people throughout the region. There are three welfare specifications based on RUM theory with independent choices. The first uses 100 choice occasions, with income \( y_i \) entering each choice occasion as annual income. The second specification is also based on 100 choice occasions, but uses per-period income (\( y_i/100 \)) in each choice occasion. The third specification is based on 200 choice occasions and uses per-period income (\( y_i/200 \)) in each choice occasion.

Two additional specifications in Table 2 do not rely on RUM and independent choices, but are instead based on the demand-system approach to repeated logit. The first is a demand-system model using annual income and 100 choice occasions. It is identical to the first specification using independent choices, except that welfare is now calculated as the area traced out by a shift in income-compensated demand functions (the details of all welfare calculations are explained below). A second demand-system model is estimated using annual income and 200 choice occasions.

For each of the five specifications, Table 2 shows three welfare measures: consumer surplus, equivalent variation, and compensating variation. Consumer surplus ignores income effects and is calculated using methods described below, except that any adjustments to income are made with the coefficient \( \beta_i \) set to zero. EV and CV account for income effects and are calculated using the estimated \( \beta_i \) from Table 1. EV and CV are presented as deviations from consumer surplus. All measures of the welfare change are negative, reflecting a decline in the quality (and value) of recreation trips associated with a hypothetical narrowing of Mid-Atlantic beaches.

A. Welfare Calculations

In the specifications based on RUM with independent choices, welfare was calculated separately for each choice occasion using simulation methods described in McFadden (1999) and Herriges and Kling (1999). These methods are based on the RUM expression \( U_{ij} = V(y_i,x_j,z_i) + \varepsilon_{ij} \) and involve: 1) taking a draw from the distribution of random errors \( \varepsilon_{ij} \), representing unobserved preferences associated with a given individual and choice; 2) adjusting the attributes \( x_j \) so that \( V_{ij} \) becomes \( V'_{ij} \), representing a decline in beach quality (the narrowing of beaches); and 3) adjusting income \( y_i \), for a given individual and choice, to the point where utility for the income-adjusted \( \max_i \{V'_{ij} + \varepsilon_{ij}\} \) is equal to \( \max_i \{V_{ij} + \varepsilon_{ij}\} \) (compensating variation) or the income-adjusted \( \max_i \{V'_{ij} + \varepsilon_{ij}\} \) is equal to \( \max_i \{V_{ij} + \varepsilon_{ij}\} \) (equivalent variation). 2 This procedure was repeated and averaged over 1,000 random draws for each choice occasion and each individual. The results were extrapolated to the full population to generate the estimates in Table 2.

Methods for calculating EV and CV using the area under income-compensated demand functions are described in Bullock and Minot (2006). The procedure is performed for each individual in the sample and then extrapolated to the full population. The first step involves finding the area under the income-compensated demand functions for the system of \( J \) sites using numerical integration methods first described in Vartia (1983). This can be done one site at a time, in any arbitrary order, and is conditional on a given quality level (for example, \( V_{ij} \) or \( V'_{ij} \)). Starting with a given site \( j \), the area under the demand function between the current price an individual faces and the individual’s choke price is traced out by a series of incremental changes in the price of \( j \). For each small change in price, the area traced out can be approximated as the change in price multiplied by the average of two quantities: the initial demand for trips to site \( j \) with no price change, and the new demand for trips after the price change. The change in area is equal to the consumer surplus loss resulting from the
change in price. As compensation for the change in price, income $y_i$ in (3) is then increased by an amount equal to the change in area, so that the demand functions are income-compensated. The incremental prices changes and income adjustments are repeated until the choke price for site $j$ is reached (the choke price can be any price high enough that demand is essentially zero). For subsequent sites, demand is conditioned on prices set at the choke point for all previous sites. The sum of the areas traced out by the incremental price changes across all sites is approximately equal to the area under the system of income-compensated demand functions. The approximation is arbitrarily close to the true value as the size of the price changes is made arbitrarily small.

The change in area reflecting the quality change from $V_{ij}$ to $V'_{ij}$ can then be approximated in an analogous way, as described in Bullock and Minot (2006). First the change in area for a small change in quality is approximated. This is just the difference between two areas calculated using the Vartia methods described above, the first area based on demand equations in (3) with no change in quality and the second area based on the same demand equations, but with an incremental change in quality. Income $y_i$ is then adjusted by an amount equal to the change in area to offset the incremental change in quality. The process is repeated for a series of small changes in quality, each time starting with the income-adjusted demand functions from the previous step. The sum of incremental adjustments to $y_i$ is equal to the income-compensated welfare change. If the starting point is $V_{ij}$, the result is a measure is CV; if the starting point is $V'_{ij}$, the result is a measure of EV.

B. Welfare Results

As described in the first section of this article, there are two key problems with welfare calculations when repeated choices are treated as independent. The first is that income adjustments to offset changes in quality are applied to only a single choice, rather than to the series of choices associated with a given individual. When the influence of income adjustments across an individual’s repeated choices is ignored, we expect the magnitude of income effects to be attenuated. This is evident in the first specification in Table 2, using independent choices, annual income, and 100 choice occasions. For this specification the estimated deviation between CS and (income-compensated) CV is -78.4, considerably smaller than the estimates of -639.1 and -695.0 for the two demand-system specifications.

The second key problem in the treatment of independent choices is the use of per-period income instead of annual income. In Table 2, the second model with independent choices is the same as the first, except that annual income is replaced by per-period income ($y_i$ is replaced by $y_i/100$). In the second model, the deviation between CS and CV increases by a factor of 100 relative to the first model (from -78.4 to -7,857.1). The implication is that income effects are considerably more significant in the second model compared to the first, but this simply reflects the rescaling of the income variable: when income is divided by 100, there is a 100-fold increase in the estimated coefficient on income. The arbitrary nature of the results is further highlighted by the third model, in which the number of choice occasions is increased from 100 to 200, and the welfare estimates change again, this time by a factor of two.

In the demand-system specifications, welfare estimates agree with expectations in a variety of ways. First, estimates of the deviation between CS and CV for the demand-system specifications lie between the estimate obtained from the first model with independent choices, which is expected to be too small, and estimates obtained from the second and third models, which are arbitrarily large. Second, the demand-system estimates remain relatively stable across alternative specifications using 100 or 200 choice occasions, indicating that arbitrary decisions about the way the model is specified have no substantive effect on
predicted welfare. The demand-system estimates are also consistent with prior theoretical expectations. For a decline in the quality of a normal good, CV should be a larger negative number than CS, and EV should be a smaller negative number than CS (for the intuition, see Freeman 2003). This relationship is evident in each of the demand-system models in Table 2.4 Finally, the three welfare measures CS, CV, and EV are quite close in size for the demand-system models, which is expected given that expenditures on beach trips represent a small portion of income for most people (Willig 1976).

V. Conclusions

This article proposed a supporting theory for logit models with repeated choices based on Slutsky symmetry of the associated demand equations. There are several implications for researchers. The first is that a logit model with income effects of the form given in (2) and (3) is consistent with utility-maximizing behavior, and can be used to generate theoretically appropriate Hicksian welfare measures. Utility consistency is also assured for commonly used logit models without income effects, such as Montgomery and Needelman (1997), Parsons et al. (1999), and MacNair and Desvousges (2007). These models are equivalent to (2) and (3) but with the income coefficient set to zero. Researchers often allow for the nesting of groups of choice alternatives that share common characteristics (e.g. Morey et al. 2002), and it is straightforward to show that these nested models are also utility consistent based on Slutsky symmetry. Finally, in all these specifications utility consistency is maintained when random coefficients are introduced (e.g., Herriges and Phaneuf 2002). This is because randomization simply expands the logit specification into a distribution of models, each of which has the same utility properties as the fixed-coefficient model of the same structure.
BIBLIOGRAPHY


The term $\beta y_i$ in (2) represents the influence of income on the preference for non-recreation activities relative to the preference for recreation trips. This type of income-related shift in demand is a common way to represent income effects in a demand-system context. In a random-utility context, it would be more natural to include the income term in all choice utilities. This would require subtracting $\beta y_i$ from each term in (2), an innocuous transformation that has no effect on utility rankings. The transformed utilities are $V_{ij} = (-\beta_p) (y_i - p_{ij}) + \beta_x x_j$ and $V_{i0} = V_0 + (\beta_y - \beta_p)y_i + \beta_z z_i$. Income effects are then represented as a coefficient on income that varies across choice alternatives, a commonly recognized way to introduce income effects in RUM models (McFadden 1999; Morey et al. 2003).

For these calculations the elements of $V_{ij}$ must be rearranged as described in the previous note.

Table 1 shows only the coefficients for models with annual income. For the models with per-period income, all coefficients remain the same except for the coefficient on income. The income coefficient increases by a factor of 100 for the model with $y_i/100$, and increases by a factor of 200 for the model with $y_i/200$.

For a single discrete choice, the relationship between CS, CV, and EV need not involve strict inequalities even when income effects are present. This is evident in Table 2 when choices are treated as independent (single) discrete choices.
### Table 1. Model Parameters

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>100 choice occasions</th>
<th></th>
<th>200 choice occasions</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Estimate</td>
<td>t-statistic</td>
<td>Estimate</td>
<td>t-statistic</td>
</tr>
<tr>
<td><strong>Travel cost ($\beta_p$)</strong></td>
<td>-0.03</td>
<td>-49.66</td>
<td>-0.03</td>
<td>-49.48</td>
</tr>
<tr>
<td><strong>Beach characteristics</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Length of shoreline</td>
<td>0.13</td>
<td>4.56</td>
<td>0.13</td>
<td>4.57</td>
</tr>
<tr>
<td>Amusement park, rides, or games available</td>
<td>1.03</td>
<td>15.70</td>
<td>1.03</td>
<td>15.69</td>
</tr>
<tr>
<td>Private or limited access</td>
<td>-0.82</td>
<td>-11.82</td>
<td>-0.82</td>
<td>-11.81</td>
</tr>
<tr>
<td>Federal park, state park, or wildlife refuge</td>
<td>0.32</td>
<td>3.70</td>
<td>0.32</td>
<td>3.71</td>
</tr>
<tr>
<td>Beach width greater than 200 feet</td>
<td>0.55</td>
<td>11.28</td>
<td>0.55</td>
<td>11.26</td>
</tr>
<tr>
<td>Beach width less than 75 feet</td>
<td>-0.27</td>
<td>-2.88</td>
<td>-0.27</td>
<td>-2.86</td>
</tr>
<tr>
<td>Beach in Atlantic City</td>
<td>1.21</td>
<td>15.80</td>
<td>1.21</td>
<td>15.78</td>
</tr>
<tr>
<td>Recognized as good for surfing</td>
<td>-0.23</td>
<td>-4.82</td>
<td>-0.23</td>
<td>-4.80</td>
</tr>
<tr>
<td>Located in developed area</td>
<td>0.65</td>
<td>13.83</td>
<td>0.65</td>
<td>13.81</td>
</tr>
<tr>
<td>Beach includes a park area</td>
<td>0.51</td>
<td>7.94</td>
<td>0.51</td>
<td>7.85</td>
</tr>
<tr>
<td>Facilities such as showers and bathrooms available</td>
<td>0.18</td>
<td>3.12</td>
<td>0.18</td>
<td>3.15</td>
</tr>
<tr>
<td>Beach located in New Jersey</td>
<td>-1.23</td>
<td>-15.96</td>
<td>-1.22</td>
<td>-15.71</td>
</tr>
<tr>
<td><strong>Respondent characteristics</strong></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Works full time</td>
<td>-1.03</td>
<td>-12.86</td>
<td>-0.99</td>
<td>-12.76</td>
</tr>
<tr>
<td>Works part time</td>
<td>-0.42</td>
<td>-5.46</td>
<td>-0.41</td>
<td>-5.45</td>
</tr>
<tr>
<td>Retired</td>
<td>-0.81</td>
<td>-9.36</td>
<td>-0.75</td>
<td>-9.02</td>
</tr>
<tr>
<td>Works at home</td>
<td>-0.72</td>
<td>-8.05</td>
<td>-0.69</td>
<td>-8.03</td>
</tr>
<tr>
<td>Owns property on a beach</td>
<td>-1.10</td>
<td>-22.42</td>
<td>-1.03</td>
<td>-21.87</td>
</tr>
<tr>
<td>Age</td>
<td>0.01</td>
<td>5.00</td>
<td>0.01</td>
<td>4.44</td>
</tr>
<tr>
<td>Education - high school only</td>
<td>-0.64</td>
<td>-14.12</td>
<td>-0.61</td>
<td>-13.84</td>
</tr>
<tr>
<td>Education - some college</td>
<td>-0.32</td>
<td>-7.43</td>
<td>-0.30</td>
<td>-7.31</td>
</tr>
<tr>
<td>Race - white</td>
<td>-0.46</td>
<td>-10.29</td>
<td>-0.45</td>
<td>-10.19</td>
</tr>
<tr>
<td>Two-income household</td>
<td>0.25</td>
<td>6.37</td>
<td>0.24</td>
<td>6.23</td>
</tr>
<tr>
<td>Head of household</td>
<td>-0.19</td>
<td>-3.53</td>
<td>-0.18</td>
<td>-3.29</td>
</tr>
<tr>
<td>Household size</td>
<td>-0.04</td>
<td>-3.14</td>
<td>-0.05</td>
<td>-3.31</td>
</tr>
<tr>
<td>Self-employed</td>
<td>-0.14</td>
<td>-2.13</td>
<td>-0.10</td>
<td>-1.67</td>
</tr>
<tr>
<td>Male</td>
<td>-0.21</td>
<td>-5.84</td>
<td>-0.20</td>
<td>-5.81</td>
</tr>
<tr>
<td>Annual household income ($0,000) ($\beta_y$)</td>
<td>-0.14</td>
<td>-30.66</td>
<td>-0.13</td>
<td>-30.14</td>
</tr>
<tr>
<td>Constant term $(V_0)$</td>
<td>5.50</td>
<td>38.91</td>
<td>6.18</td>
<td>44.97</td>
</tr>
<tr>
<td>Scale parameter $(\sigma)$</td>
<td>1.54</td>
<td>37.41</td>
<td>1.60</td>
<td>37.23</td>
</tr>
</tbody>
</table>
## Table 2. Annual Welfare Measures ($000)

<table>
<thead>
<tr>
<th>Welfare Specification</th>
<th>Consumer Surplus</th>
<th>Deviation from Consumer Surplus</th>
<th>Equivalent Variation</th>
<th>Compensating Variation</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>RUM with Independent Choices</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Annual income, 100 choice occasions</td>
<td>-1,238,653 (221,215)</td>
<td>0.0</td>
<td>-78.4 (27.7)</td>
<td></td>
</tr>
<tr>
<td>Per-period income, 100 choice occasions</td>
<td>-1,238,653 (221,215)</td>
<td>0.0</td>
<td>-7,857.1 (3,538)</td>
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</tr>
<tr>
<td>Per-period income, 200 choice occasions</td>
<td>-1,235,745 (221,128)</td>
<td>0.0</td>
<td>-15,389.4 (6,729)</td>
<td></td>
</tr>
<tr>
<td><strong>Logit Demand System</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Annual income, 100 choice occasions</td>
<td>-1,239,283 (221,314)</td>
<td>630.1 (213.2)</td>
<td>-639.1 (215.5)</td>
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</tr>
<tr>
<td>Annual income, 200 choice occasions</td>
<td>-1,236,897 (221,854)</td>
<td>689.0 (231.1)</td>
<td>-695.0 (232.0)</td>
<td></td>
</tr>
</tbody>
</table>

Note: Standard errors (in parentheses) were calculated based on 100 draws from the covariance matrix of model parameters.