Modelling Stochastic Route Choice with Bi-objective Traffic Assignment

Judith Y T Wang∗, Matthias Ehrgott†

Abstract

In this paper, we propose a novel approach to model stochastic route choice in a tolled road network. First of all, we assume that all users have two objectives: (1) minimise travel time; and (2) minimise toll cost. Users are all rational in the sense that given a choice set, they will only choose one of the efficient paths. This will result in a bi-objective user equilibrium (BUE) condition such that traffic arranges itself in such a way that no individual trip maker can improve either his/her toll or travel time or both without worsening the other objective by unilaterally switching routes.

We assume further that users have different preferences in the sense that for any given path with a specific toll, there is a limit on the time that an individual would be willing to spend. Each individual can have his/her own preference represented by this indifference function between toll and time. Time surplus is defined as the maximum time minus the actual time. Given a set of paths, the one with the highest (or least negative) time surplus will be the preferred path for the individual. As a result, for a specific origin-destination (O-D) pair, each individual can have a different preferred path, even though all individuals are considering the same choice set.

In this way, based on the distribution of individual indifference curves, we can deduce the bi-objective equilibrium solution satisfying the time surplus maximisation bi-objective user equilibrium (TSMaxBUE) condition. That is, for each O-D pair, all individuals are travelling on the path with the highest time surplus value among all the efficient paths between this O-D pair.

The philosophy behind the TSMaxBUE model is to overcome the restrictions that came with the two most commonly applied methods in tolling analysis, namely, user equilibrium (UE) and stochastic user equilibrium (SUE). UE assumes that all individuals behave the same way, i.e. to minimise generalised cost. The difference in individual preferences is modelled by creating user classes with different values of time (VOT). As a result, UE is restricted by the assumption that all users with the same VOT behave in exactly the same way, which can be dictated by a generalised cost function. SUE assumes that drivers choose their preferred routes based on perceived costs on different routes. Some formulations of this model come with the well known independence of irrelevant alternatives (IIA) property and assumptions on the probability density function of the error term of the utility function. Both the IIA property and assumptions on the error term have imposed limitations on the capability to replicate travel behaviour in reality. The model proposed here can overcome all these difficulties by introducing bi-objective traffic assignment and an indifference function between toll and time which can vary between individuals with no restrictions.

∗Dr Judith Wang, Research Fellow, The Energy Centre, The University of Auckland Business School, email: j.wang@auckland.ac.nz
†Professor Matthias Ehrgott, Department of Engineering Science, The University of Auckland, email: m.ehrgott@auckland.ac.nz
1 Introduction

The last stage of a conventional four-stage transport planning model, traffic assignment, is essentially modelling the route choice behaviour of travellers and their interactions. Whether a traffic assignment model can realistically represent travel behaviour is, therefore, dependent on the behavioural assumptions behind the route choice model. In tolling analysis, there are basically two approaches in practice as described in Florian (2006): (1) models based on generalised cost path choice; and (2) models based on explicit choice of tolled facilities. These two approaches follow the principles of the two classic traffic assignment models in the literature, namely, the user equilibrium (UE) model and the stochastic user equilibrium (SUE) model.

Wardrop (1952) defined user equilibrium as:

“No user can improve his travel time by unilaterally changing routes.”

This is known as Wardrop’s first principle which has two key assumptions: (1) all users have the same objective, i.e. to minimise travel time or generalised cost; and (2) users have perfect knowledge of the network, i.e. they know the travel times that would be encountered on all available routes between their origin and destination. The second assumption is considered to be a strong assumption which might not be realistic. Dial (1971) was the first to introduce a probabilistic assignment concept to address this problem. He proposed a probabilistic multipath traffic assignment model based on the following functional principles:

1. The model gives all efficient paths between a given origin and destination a non-zero probability of use, while all inefficient paths have a probability of zero.
2. All efficient paths of equal length have an equal probability of use.
3. When there are two or more efficient paths of unequal length, the shorter has the higher probability of use.

The meaning of ‘efficient’ paths in Dial’s model is defined as a path that does not backtrack, i.e. as it progresses from node to node, it always gets further from the origin and closer to the destination. Every link in an efficient path has its initial node closer to the origin than its final node and its final node closer to the destination than its initial node. In this manner, the set of ‘efficient’ paths can be considered as the reasonable choices. By introducing diversion curves, Dial (1971) incorporated the logit function into his model which enables the solution to be expressed in explicit form. However, congestion effects have not been considered in this model as link travel time is assumed to be constant.

SUE was developed by Daganzo and Sheffi (1977) based on variation of the first assumption of Wardrop’s first principle by considering the objective as minimising the perceived cost which is modelled as a stochastic function rather than the static generalised cost function. Daganzo and Sheffi (1977) defined stochastic user equilibrium as:

“No user can improve his perceived travel time by unilaterally changing routes”

In order to translate this SUE equilibrium condition into its mathematical definition, Daganzo and Sheffi introduced a user’s perceived travel time function on route $k$, $T_k$, which has two components as follows:

$$T_k = t_k + \epsilon_k,$$ (1)
where $t_k$ is the systematic component which is the measured travel time on route $k$; and $\epsilon_k$ is an error term representing the random component which varies from user to user.

Here $\epsilon$ is randomly distributed with a mean value of zero. Thus,

$$E(T_k) = t_k.$$  \hfill (2)

Every user then evaluates the travel time on all routes and selects the route $k_{\text{min}}$ with the minimum perceived travel time, i.e.

$$T_{k_{\text{min}}} \leq T_k \text{ for all } k \neq k_{\text{min}}.$$  \hfill (3)

The mathematical conditions for SUE within this modelling framework are formally defined in Daganzo and Sheffi (1977). The assumption on the distribution of the error term, $\epsilon_k$, varies. The most commonly used distributions are Gumbell and normal distributions, known as the logit and probit models, respectively. The assumption of the error term following Gumbell/normal distributions is the key linkage of Dial (1971)'s probabilistic model to Discrete Choice Models, which led to further development of SUE traffic assignment models that appeared later in the literature such as Fisk (1980)'s logit-based model and Sheffi and Powell (1982)'s probit model. It is important to note that in order to take congestion effects into consideration, travel time should be flow-dependent. Fisk (1980) was the first to consider the effect of congestion in a stochastic manner, as travel time is considered to be independent of traffic flow in the previous models (Dial, 1971; Daganzo and Sheffi, 1977).

A disadvantage of a probit model is well known as intensive computational effort requiring Monte Carlo techniques. Logit models have their weaknesses but a very important advantage of having a closed form solution. Thus, the most commonly used stochastic traffic assignment model for toll analysis is a logit-based model as described in Florian (2006). The key weakness of the most commonly used logit-based model is the validity of the assumption of independence of irrelevant alternatives (IIA), which can be stated as:

“Where any two alternatives have a non-zero probability of being chosen, the ratio of one probability over the other is unaffected by the presence or absence of any additional alternative in the choice set (Luce and Suppes, 1965).”

When it comes to modelling path choice, the IIA property can be easily violated because of extensive overlapping of possible paths in a choice set for the same origin-destination (OD) pair. Over the last two decades, there were extensive developments in stochastic route choice models trying to address this weakness. Prashker and Bekhor (2004) provide a comprehensive review of the developments. Since the perceived cost function has two components as shown in Equation (1), this problem can be addressed by tackling either the systematic or the error component. In principle, the technique being used is to make adjustments to these two components such that the resulting solution reflects reality better. Prashker and Bekhor (2004) classified the techniques into three categories: (1) modifications of the basic multinomial logit (MNL) model, such as C-logit and path-size logit (PSL); (2) generalised extreme value (GEV) models, such as paired combinatorial logit (PCL) and cross-nested logit (CNL); and (3) logit kernel (LK) or mixed logit models. The first category adjusts the systematic component while the second and the third adjust the error component.

In this paper, we propose a novel approach to model stochastic route choice in a tolled network. We extend our work in Wang et al. (2010) on bi-objective traffic assignment to incorporate the capability to model the differences between individuals in terms of their willingness to pay. First of all, we assume that all users have two objectives: (1) minimise travel time; and (2) minimise
Users are all rational in the sense that they will only choose one of the *efficient* paths. Efficient paths are defined as the set of paths for each O-D pair for which neither time nor travel time can be improved without worsening the other (Wang *et al.*, 2010). According to this definition, at equilibrium, all the used paths between a given O-D pair are *efficient*. We define *bi-objective user equilibrium* (BUE) as follows:

“Under *bi-objective user equilibrium* conditions traffic arranges itself in such a way that no individual trip maker can improve either his/her toll or travel time or both without worsening the other objective by unilaterally switching routes.”

Dial (1979) is one of the first to introduce bi-objective in traffic assignment. According to BUE, when we consider time and toll cost separately, there is no need to add them up as generalised cost. However, in Dial’s model (Dial, 1979, 1996, 1997), a simplification was made by adding time and toll cost in a linear choice function, which is essentially the same as the generalised cost function, but with a probabilistic component by assuming that the value-of-time (VOT) follows a certain probability density function. As discussed in Wang *et al.* (2010), Dial’s approach can easily miss out some efficient paths. In Wang *et al.* (2010), we developed heuristics to find BUE solutions without missing efficient paths. It is clear that according to the BUE definition, there would be many possible equilibrium solutions rather than the conventional static UE solution. Given there are so many possible equilibrium solutions satisfying the BUE condition, we must further develop this model to incorporate the consideration of individual preferences in order to be able to replicate their route choice behaviour more realistically.

There is no doubt that route choice behaviour in a tolled road network is stochastic in nature since individuals might not choose the shortest path for all sorts of reasons and the willingness to pay would vary among individuals. As discussed above, probabilistic models such as Dial (1979)’s or the logit-based SUE traffic assignment models such as Fisk (1980)’s all possess some deficiencies. The philosophy behind the proposed model is to overcome these difficulties, including the possibility of missing efficient paths in Dial (1979)’s model and the limitations induced by the IIA property of the logit-based SUE traffic assignment model, by introducing an indifference function which can vary between individuals with no restrictions. As with any models, there are, however, some key assumptions to be made:

1. Users are all rational in the sense that they will only choose one of the *efficient* paths.
2. Users have different *preferences* which can be represented by an indifference function between toll and time. Users’ behaviour as represented by this indifference function is rational, i.e. the maximum time that a user is willing to spend will always be shorter for higher toll.
3. Preferences among users vary in the sense that their preferred paths can be different, even though they are considering the same choice set.

With this new approach, each individual will only choose from a reasonable choice set and choose according to his/her own preference.

## 2 The Time Surplus Maximisation Choice Model

### 2.1 The Indifference Function

We assume that for each O-D pair, each user has his/her individual indifference function between toll and time. For any given path with a specific toll, there is a limit on the time that an individual would be willing to spend. Time surplus is defined as the time that the user would be willing
to spend minus the actual travel time. The time surplus for a path can be positive or negative. Given a choice set of paths, the one with the highest time surplus will be the preferred path for the individual.

A positive time surplus value can be viewed as virtually the pleasure for an individual obtained from choosing this path, whereas a negative time surplus value can represent an unfavourable choice and the magnitude of this path being disliked. One would expect that given a set of efficient paths with both positive and negative time surplus values, only the ones with positive time surplus values will be considered. In other words, the set of efficient paths with positive time surplus values would naturally be the reasonable choice set.

For example, an individual with a convex indifference curve as shown in Figure 1 will only consider the two paths that have positive time surplus: (1) Toll=20; and (2) Toll=0. Among these two, the one with Toll=20 is considered more attractive as the time surplus value is higher. On the other hand, an individual with a concave indifference curve as shown in Figure 2 will consider all three paths. In this case, again the path with Toll=20 is considered most attractive.

There is, however, the possibility that all the efficient paths have negative time surplus values for a user who is both unwilling to pay and to spend time. In that case, we will have to assume either this user would not travel at all or will have to make a choice based on the negative values. In this paper, we assume that the total demand is inelastic and hence the user will choose the path with the least negative time surplus value.

Based on the distribution of individual indifference curves, we can derive the distribution of the maximum time users are willing to spend on different paths between the same O-D pair. For example, one possible maximum time distribution for the three routes of Figures 1 and 2 is shown in Figure 3. Given this distribution, it also means that an equilibrium solution would have travel times on the left of the red dotted line in Figure 4, since the red dotted line represents the maximum travel time on each path that any individual will be willing to spend.

### 2.2 The Time Surplus Maximisation BUE Condition

The Time Surplus Maximisation BUE condition is defined as:
“Under the *Time Surplus Maximisation condition* traffic arranges itself in such a way that no individual trip maker can improve his/her time surplus by unilaterally switching routes,”

or alternatively

“Under the *Time Surplus Maximisation condition* all individuals are travelling on the path with the highest time surplus value among all the efficient paths between each O-D pair.”

The time surplus maximisation BUE model basically follows similar functional principles as in Dial (1971).

1. Traffic will only be assigned to *efficient* paths. Note that we define *efficient* paths differently but basically the meaning of our definition also identifies the set of reasonable choices.

2. All dominated (inefficient) paths will have zero probability of use.

3. If there are two or more efficient paths, the one with the highest time surplus will be chosen.

Because the time surplus of a dominated path (a path for which there is an alternative path whose travel time and toll is not worse than that of the dominated path, with at least one of them being strictly better) is never better than that of any efficient path dominating it, we only include *efficient* paths in the choice set which gives us a reasonable choice set. The variability between individuals in terms of willingness to pay is modelled by the indifference function which leads to their differences in behaviour.

Next we show that the time surplus maximisation condition does indeed define a BUE solution.

**Theorem 1** Every solution satisfying the time surplus maximisation condition also satisfies the bi-objective user equilibrium condition.

**Proof:** Assume the assertion is false. Then there is at least one OD-pair and one user that can switch to a different path such that they either
• reduce toll cost without increasing travel time, or
• reduce travel time without increasing toll cost.

In the former case the reduced toll cost and the rationality assumption regarding the indifference function imply that the user would accept a longer maximum travel time on the cheaper route. But because travel time does not increase on the new route, their time surplus would increase, contradicting the time surplus maximisation condition.

In the latter case, since the toll is not increasing, the maximum time they are willing to travel does not decrease, hence the shorter travel time implies a bigger time surplus, once again contradicting the time surplus maximisation condition.

\[ 2.3 \quad \text{User Equilibrium Solution} \]

We denote by \( Q_{rs} \) the number of trips from origin \( r \) to destination \( s \), by \( x_a \) the traffic flow on link \( a \) (veh/time unit), and by \( t_a(x_a) \) the travel time at traffic flow \( x_a \) on link \( a \). The Bureau of Public Roads (1964) function will be applied to model the relation between travel time and traffic flow:

\[
t_a(x_a) = t_0^a \left[ 1 + \alpha \left( \frac{x_a}{C_a} \right)^\beta \right], \quad (4)
\]

where \( t_0^a \) is the free-flow travel time on link \( a \), \( C_a \) is the practical capacity of link \( a \) (veh/time unit), and \( \alpha, \beta \) are function parameters (\( \alpha = 0.15, \beta = 4.0 \)).

Conventional assignment algorithms were developed based on Beckmann’s transformation of the UE conditions to a mathematical programming formulation (Beckmann et al. (1956)). This programme includes a convex (non-linear) objective function and a linear constraint set as follows:

\[
\begin{align*}
\min Z(x) &= \sum_a \int_0^{x_a} t_a(\omega) d\omega, \\
\text{subject to } &\sum_k f_{rs}^k = Q_{rs} \text{ for all } r, s, \\
&f_{rs}^k \geq 0 \text{ for all } k, r, s, \\
&\text{definitional constraints } x_a = \sum_r \sum_s \sum_k f_{rs}^k \delta_{rs} = x_a, \quad (5)\end{align*}
\]

where \( x_a, t_a(\omega) \) and \( Q_{rs} \) are as defined above, \( f_{rs}^k \) is the flow on path \( k \) connecting origin \( r \) with destination \( s \), and \( \delta_{rs}^k = 1 \) if link \( a \) is a part of path \( k \) connecting origin \( r \) with destination \( s \), and \( \delta_{rs}^k = 0 \) otherwise.

\[ 2.4 \quad \text{Time Surplus Maximisation BUE Solution} \]

Denote the maximum time individual \( i \) travelling from \( r \) to \( s \) is willing to spend for the trip as \( t_i^{\text{max}} \). We model \( t_i^{\text{max}} : \mathbb{R}_+ \to \mathbb{R}_+ \) as a strictly decreasing function such that \( t_i^{\text{max}}(\tau) \) is the maximum time user \( i \) is willing to spend for the trip from \( r \) to \( s \) if the toll is equal to \( \tau \). If \( \tau_k \) is the toll on path \( k \) and the the travel time on path \( k \) from \( r \) to \( s \) is \( t_k \) then the time surplus on path \( k \) for individual \( i \) is

\[
t_{ik}^{\text{surplus}} = t_i^{\text{max}}(\tau_k) - t_k. \quad (9)
\]

Note that \( r \) and \( s \) are omitted in the equation for simplicity. Note that, because we assume that the \( t_i^{\text{max}} \) function depends on the O-D pair and \( s \), but is independent of the specific path
from \( r \) to \( s \), this choice function is path-based and cannot be written as the sum of link functions. Hence we cannot use Beckmann’s argument to derive an equivalent optimisation problem analogous to Equations (5) – (8) in the case of user equilibrium.

While in general networks, the path choice function in Equation (9) cannot be written in a link-based form, the situation is simpler as long as paths and links are identical. In that case, the time surplus function in Equation (9) for path \( k \) is also a link-based function. In such a network, we incorporate individual preferences given by the indifference curves \( t_{\text{max}}^i \) of all the individuals \( i \) by deriving a probability density function of the maximum time willing to spend on path \( k \) as \( f (t_{\text{max}}^k) \).

For networks in which links are equivalent to paths, i.e. for each link \( a \) we have \( x_a = f_{rs}^k \) for exactly one \( r, s \) and \( k \), Equation (9) is a continuous, strictly decreasing and separable function of link flow. We can then reformulate the time surplus maximisation condition as the following concave maximisation formulation. As in the proof of the equivalence of Equations (5) – (8) to Wardrop’s user equilibrium condition, the first order necessary conditions for optimality turn out to be the equilibrium conditions.

\[
\max Z(x) = \sum_a \int_0^\infty f (t_{\text{max}}^a) \int_0^{x_a} \left[ t_{\text{max}}^a - t_a (\omega) \right] d\omega dt_{\text{max}}^a, \\
\text{subject to } \sum_k f_{rs}^k = Q_{rs} \text{ for all } r, s, \\
f_{rs}^k \geq 0 \text{ for all } k, r, s, \\
\text{definitional constraints } x_a = f_{rs}^k \delta_{rs}^k \text{ for all } a, k, r, s.
\]

Here \( \delta_{rs}^k = 1 \) if link \( a \) is the \( k \)th path from origin \( r \) to destination \( s \) and 0 otherwise. According to the argument above we have the following theorem.

**Theorem 2** Let \( x^* \) be the (unique) optimal solution of the optimisation problem of Equations (10) – (13). Then \( x^* \) satisfies the time surplus maximisation BUE condition.

Note that the formulation as illustrated in Equations (10) – (13) is a link-based formulation. This equivalence is true only for networks in which links are equivalent to paths, because the time surplus function in Equation (9) is not additive in general. Any optimisation formulation for general networks will need to be path-based rather than link-based.

### 3 A Three-link Example

#### 3.1 Network Specification

Now we consider a three-link example as shown in Figure 5 with route characteristics as shown in Table 1. Note that Route 1 is the fastest with the highest toll while Route 3 is toll free and the slowest. The total demand from \( r \) to \( s \) is fixed at 15,000 vehicles per hour. The link travel time is assumed to be a function of traffic flow following the Bureau of Public Roads (1964) function as shown in Equation (4). For simplicity, we assume the maximum time that an individual is willing to spend on each link follows a uniform distribution with the upper and lower bound values as shown in Table 2.
Table 1: Route characteristics of the three-link network.

<table>
<thead>
<tr>
<th>Route</th>
<th>Type</th>
<th>Distance (km)</th>
<th>Speed Limit (km/hr)</th>
<th>Free flow travel time (mins)</th>
<th>Toll ($)</th>
<th>Capacity (veh/hr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Expressway</td>
<td>20</td>
<td>100</td>
<td>12</td>
<td>40</td>
<td>4000</td>
</tr>
<tr>
<td>2</td>
<td>Highway</td>
<td>50</td>
<td>100</td>
<td>30</td>
<td>20</td>
<td>5400</td>
</tr>
<tr>
<td>3</td>
<td>Arterial</td>
<td>40</td>
<td>60</td>
<td>40</td>
<td>0</td>
<td>4800</td>
</tr>
</tbody>
</table>

Table 2: Maximum time willing to spend

<table>
<thead>
<tr>
<th>Route</th>
<th>Lower bound ( t^L_k )</th>
<th>Upper bound ( t^U_k )</th>
<th>Probability density ( f(t^\text{max}_k) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
<td>25</td>
<td>1/(25-10)</td>
</tr>
<tr>
<td>2</td>
<td>30</td>
<td>45</td>
<td>1/(45-30)</td>
</tr>
<tr>
<td>3</td>
<td>60</td>
<td>90</td>
<td>1/(90-60)</td>
</tr>
</tbody>
</table>

Figure 5: A Three-link Example Network
3.2 The Conventional Solutions (UE, SUE & Social Optimum)

Assuming demand is inelastic, i.e. all users must travel, the solution space for this three-link network can be represented two-dimensionally as shown in Figure 6, with contours of the total travel time. We first identified the following solutions, as shown in Figure 6, in the conventional way:

1. the UE solution without tolls;
2. the UE solution with tolls, assuming VOT being $1 per minute;
3. the SUE solution based on a multinomial logit formulation as shown in Equation (14)

\[
P_k = \frac{e^{\theta U_k}}{\sum_a e^{\theta U_a}},
\]

where \( P_k \) is the probability of path \( k \) to be chosen; \( U_k \) is the utility of choosing path \( k \); \( U_k \) is a function of the travel time \( t_k \) and toll \( \tau_k \), i.e. \( U_k = -t_a (x_a) \times VOT - \tau_k \); and \( \theta \) is the model parameter for calibration (assuming \( \theta = 0.05 \)); and

4. the Social Optimum (SO) solution, by minimising total travel time, i.e. replacing Equation (5) in the optimisation problem of Equations (5) – (8) with Equation (15)

\[
\min Z(x) = \sum_a x_a t_a.
\]

3.3 The BUE Solution Space

In order to illustrate the BUE solution space in this three-link example, we first discretise the solution space and identify the BUE solution space where the BUE equilibrium condition applies. Because tolls are independent of flow and \( \tau_1 > \tau_2 > \tau_3 \), the BUE condition is satisfied whenever

\[
t_1(x_1) < t_2(x_2) < t_3(x_3).
\]

The BUE solution space is illustrated three-dimensionally with total travel time as the third dimension in Figure 7 and two-dimensionally in Figure 8. We then examine the distribution of link flow and link travel time in this discretised BUE solution space. The boxplots of the link flow and link travel time are illustrated in Figures 9 and 10, respectively. The link travel time on the toll-free route has a range of 40 minutes to 612 minutes corresponding to a flow range of 1,000 to 15,000 vehicles per hour. The latter case corresponds to the case of putting all the demand on Route 3; the resulting solution will have a link travel time of 612 minutes on Route 3 while the link travel times on Route 1 and 2 are free-flow at 12 minutes and 30 minutes. This solution satisfies the BUE definition but obviously we would expect that someone would want to pay if the travel time is 612 minutes on the toll-free route. Observations made from this three-link example strongly support the urgent need for further specification of the equilibrium conditions to represent route choice behaviour more realistically.

3.4 The Time Surplus Maximisation BUE Solution

Now we examine the case of time surplus maximisation. We assume user preferences are modelled by the distribution of the maximum time that an individual is willing to spend on each link
Figure 6: Total travel time contours in the solution space of the three-link example

Figure 7: Three-dimensional plot of the BUE solution space

Figure 8: The BUE solution space boundary
following a uniform distribution as shown in Table 2. Following the formulation in Section 2.4, the
TSMaxBUE solution is identified as shown in Figures 6, 7 and 8 together with the conventional
solutions.

The equilibrium travel time on different routes under different equilibrium conditions are shown
in Figures 11 and 12, with the assumed distribution of the maximum time users are willing to
spend for the TSMaxBUE solution in the background. Some observations are made as follows:

1. The UE solution without tolls can be connected with a straight line and the line is vertical
   as the travel times on all three paths are equal.
2. The UE solution with tolls can be connected with a straight line. With a fixed VOT as-
   sumption, this represents a special case of the time surplus maximisation model, i.e. all users
have the same indifference function which is linear with a slope equal to VOT. This shows that UE is the most restrictive model among all the models considered.

3. As shown in Figure 12, by adjusting the calibration parameter $\theta$ in the SUE case, the range of equilibrium travel time is still very limited. Since we assume the VOT in the UE case with tolls is the same as in the SUE utility function, when the $\theta$-value is high, i.e. when $\theta = 2$ in this case, the SUE solution almost coincides with the UE solution with tolls.

4. As expected, the TSMaxBUE solution closely follows the pattern of the underlying indifference functions. Users are maximising their time surplus, i.e. the distance between the equilibrium travel time to the maximum time willing to spend, resulting in equilibrium points all on the left hand side of the upper bound dotted line.

4 Conclusion and Outlook

In this paper we have introduced a new model for route choice in tolled road networks. The model is based on the idea of bi-objective user equilibrium and microeconomic theory. The equilibrium refers to the condition that traffic will arrange itself in such a way that no user can decrease travel time, or toll, or both without worsening the other. Since bi-objective user equilibrium allows many possible solutions, not all of which are meaningful in practice, we have augmented the concept with the idea of time surplus maximisation based on microeconomic theory. This idea assumes that a user has an indifference function defining for any value of toll the maximum time he/she is willing to spend for travel between an origin and a destination. The preference of a user can be determined by the time surplus defined as maximum time willing to spend minus actual travel time. Users are rational and will choose a route with maximum time surplus among all efficient paths. We demonstrated that this model overcomes drawbacks of the earlier UE and SUE models. The indifference function can model variability among users with no restrictions while by including only efficient paths in the choice set naturally creates a reasonable choice set.

Because the time surplus maximisation equilibrium problem in general networks does not allow a link-based optimisation formulation, we will develop path equilibration algorithms (see Dafermos and Sparrow (1969); Nagurney (1993) for descriptions of such algorithms) for this problem in future research.

Moreover, in this paper we have only considered the case of inelastic demand, i.e. even users with only routes with negative time surplus in their choice sets will have to choose a route (the one with least negative time surplus) and travel. It is natural to extend the model to the elastic case, where users may not travel if their time surplus is negative on all efficient paths. In this case we would look at replacing the function of Equation (9) with $t_{ik}^{\text{surplus}} = [t_{i}^{\text{max}}(\tau_k) - t_k]^+$, i.e. either positive time surplus or 0 if $t_{i}^{\text{max}}(\tau_k) - t_k$ is negative, as the route choice function. This complicates analysis considerably and is a topic of current research.

5 Acknowledgments

The authors would like to acknowledge Professor Anthony Chen of Utah State University for his invaluable comments on an earlier version of the paper. This research was partially supported by the Marsden Fund, grant number 9075 362506.
References


